THE EMERGENCE OF UNSATURATED SOIL MECHANICS

The Fourth Spencer J. Buchanan Lecture

by

Professor Delwyn G. Fredlund

Friday, November 8, 1996

Lecture Room A
Clayton Williams, Jr. Alumni Center
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Texas A&M University
College Station, Texas

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ABSTRACT

The first ISSMFE conference (International Society for Soil Mechanics and Foundation Engineering) in 1936 provided a forum for the establishment of principles and equations for what has been come to be known as saturated soil mechanics. A significant number of research papers were presented on unsaturated soil behavior at the first ISSMFE conference but it would take 3 to 4 decades before a sound theoretical basis would emerge for the practice of unsaturated soil mechanics. It took visionary, pioneer engineers such as Professor Spencer Buchanan to organize the international series of conferences on expansive soils. This series of conferences would provide a focused forum for the later development of the broader discipline of unsaturated soil mechanics.

Theories and formulations have been developed for all aspects of the behavior of unsaturated soils. The behavior of saturated soils can now be shown to be a special case of the more general mechanics for saturated/unsaturated soils. The scope of saturated/unsaturated soil mechanics is illustrated in this paper through a series of visualization aids and diagrams.

Man lives in direct contact with the vadose zone (i.e., the unsaturated zone). The upper boundary of the vadose zone (i.e., the ground surface) is subjected to a flux type boundary condition for many of the problems faced by geotechnical engineers. The atmosphere, down through the vadose zone to the soil below the groundwater table should be viewed as a continuum. A generalized mechanics for soils which is applicable for both saturated soils and unsaturated soils is necessary for dealing with the soil of the subterranean zone in a consistent manner (i.e., the saturated zone and the unsaturated zone should not be treated in isolation). There is also a need to treat the atmosphere and the soils as a unit. The soil-water characteristic curve has been shown to be a key soil function which can be used to approximate the behavior of unsaturated soils.

PREAMBLE TO THE SPENCER BUCHANAN LECTURE

Let me start by expressing my appreciation to the Spencer Buchanan Lecturer committee for selecting me to deliver the 1996 Spencer Buchanan lecture. I feel honored to have been asked to fulfil this task.

In 1964, I completed my master's degree at the University of Alberta and went to work for a consulting engineering firm in Alberta, Canada. My master's thesis was an extension of my summer research with the National Research Council of Canada. The work had involved studies on the behaviour of unsaturated, expansive soils in western Canada. I had only worked for the Alberta-based consulting firm for a few months when I read that the First International Conference on Expansive Soils would be held at Texas A & M University, College Station, Texas (1965). I had never heard of Texas A & M but I knew that I wanted to go to that conference. The consulting firm paid my way to that conference and it was at that time that I first met Professor Spencer Buchanan. I was a young graduate from university and the thing that I remember most about Spencer Buchanan was his willingness to spend time talking to a young graduate engineer who did not know much about anything. However, I knew that I was interested in expansive soil behavior and I realized that I had found a very busy man who still had time to talk to me.

Later I would return to Texas A & M for the Second International Conference on Expansive Soils (1969) and again renew my acquaintance with Professor Buchanan. During Professor Buchanan's opening remarks to the conference, he said:

"It was found at the conclusion of the First Conference, that the profession was beginning to recognize and appreciate problems resulting from expansive clay soils. The consensus of the conferees of the First Conference was that firm conclusions could not be developed because of the limited basic information available at that time. It served to inspire engineers of the world to extend their research of this problem and to assemble factual information to serve as a basis for establishing criteria for the accomplishment of solutions sorely needed".

One of the engineers who had been "inspired" was myself. It was at this Second Conference (1969) that I would present my first research paper. Little did I realize that I would go on to attend all of the subsequent conferences on expansive soils and then see the change of the name to the Unsaturated Soils Conferences.

It was at these early conferences on expansive soils that I realized that while the first ISSMFE conference (International Society for Soil Mechanics and Foundation Engineering) in 1936, had provided a forum for the establishment of principles and equations for saturated soils, similar principles and equations were lacking for unsaturated soils such as expansive soils. It would still take 3 to 4 decades, along with conferences such as the expansive soils series spearheaded by Professor Buchanan, before a sound, theoretical basis would emerge for the practice of unsaturated soil mechanics. It took visionary, pioneer engineers such as Professor Spencer Buchanan to organize the international series of conferences on expansive soils. The international series of conferences on expansive soils has provided a focused forum for the later development of the broader discipline of unsaturated soil mechanics.

INTRODUCTION TO THE CASE FOR UNSATURATED SOIL MECHANICS

In the past, theoretical soil mechanics has dealt mainly with saturated soil below the groundwater table. The soil above the groundwater table was mostly dealt with using empirical formulations. A rational, scientific basis was lacking in dealing with the mechanics of the unsaturated soil media. A scientific basis is essential for problems to be properly understood and for the problems to be handled in a rational and consistent manner. In saturated soil mechanics, the principle of effective stress provided the key element for understanding the behavior of the saturated soil. The effective stress, $(\sigma - u_w)$, has been proven to be an adequate variable to define the stress state of the saturated soil. There is a need to use a similar state variable approach in the understanding of the behavior of unsaturated soils. It will be shown that there can be one generalized theory of soil mechanics whose concepts are applicable to both saturated and unsaturated soils.

In this paper, a more all-encompassing repre-sentation of the discipline of a saturated/unsaturated soil mechanics is illustrated through a series of visualization aids. An attempt is made to use simplicity in describing concepts and explaining example situations. The visualization aids may not be drawn to scale and may appear to be exaggerated situations.

THE TERM, SATURATED/UNSATURATED SOIL MECHANICS

The term, Saturated/Unsaturated Soil Mechanics, is used in the sense that the theories and formulations apply to the high majority of soil conditions encountered in engineering practice. Saturated/Unsaturated Soil Mechanics applies to soils near the ground surface as well as those at greater depths. It applies to soils above the water table as well as those below the water table. It applies to unsaturated soils and saturated soils alike using the same general theories and formulations. It can be shown that theories and formulations which embrace the unsaturated portion of a soil profile have saturated soil behavior as a special case (i.e., there is one unified theory for soil mechanics).

NEED FOR A SATURATED/UNSATURATED MECHANICS OF SOILS

A large portion of the world population is found in the arid regions of the world where the groundwater table is deep. There appears to be a strong correlation between the arid regions and the population density (Fig. 1). The 10 - 40 Window of the world is defined by $+10^{o}$ and $+40^{o}$ latitude north and 10^{o} and 40^{o} longitude. This window contains approximately 3.1 billion people or 60% of the population of the world and also contains 60% of the countries of the world.

The drier climatic regions have become increasingly aware of the uniqueness of their regional soil mechanics problems. In recent years there has also been a shift in emphasis in the developed regions from the behavior of engineered structures to the impacts of developments on the natural world. This shift in emphasis resulted in greater need to deal with the vadose zone. There has been an ongoing desire to expand the science dealing with soil mechanics such that it will also embrace the behavior of unsaturated soils. An all-encompassing Saturated/Unsaturated Soil

Mechanics has emerged in the developed countries and is today receiving acceptance on a global scale.

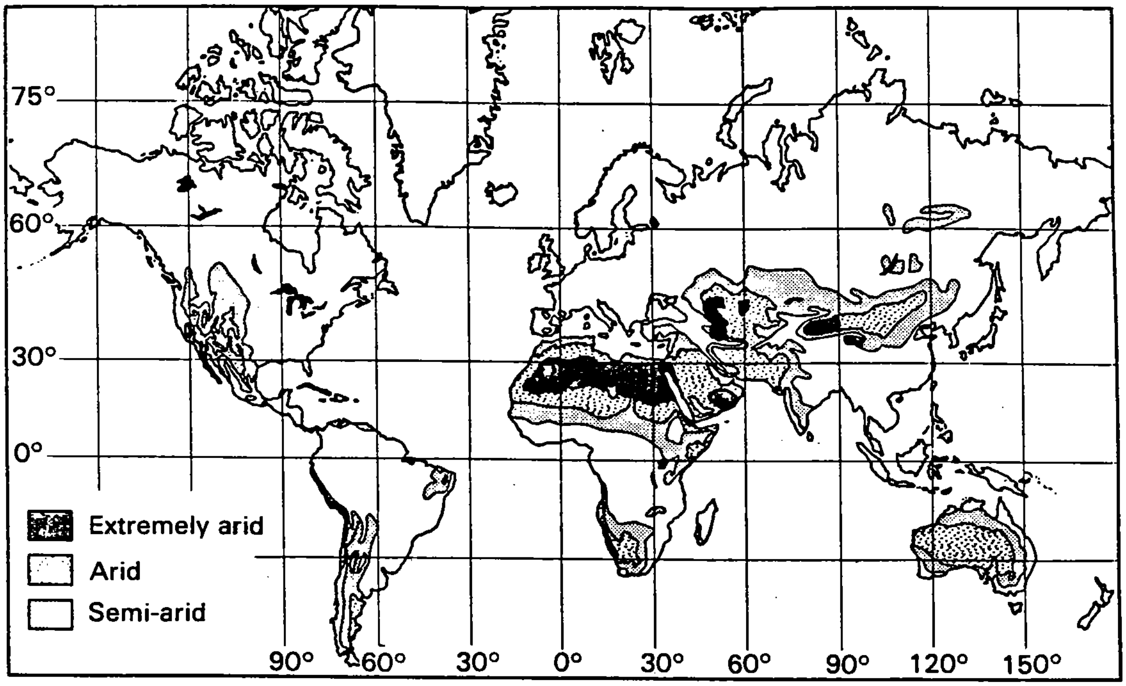


Figure 1 Map showing extremely arid, arid and semi-arid regions of the world.

There is a need to deal with the behavior of unsaturated soil beyond the utilization of empirical formulations. Empiricism generates methods of handling specific problems but does not provide proper theoretical explanations for the problems. A rational, scientific basis is also needed to teach and transfer technology such that the theories can become a part of worldwide geotechnical engineering. The effective stress variable is the key that has led to the rapid transfer of geotechnology of saturated soils around the world. Similarly, the stress state variable approach is the means of transferring unsaturated soil behavior from one country to another.

EVENTS WHICH SPURRED THE EMERGENCE OF UNSATURATED SOIL MECHANICS

The development of a rational approach to the behavior of unsaturated soils can be traced to three main events.

1.) In arid countries around the world there have been problems associated with volume changes of unsaturated soils in response to changes in the water content of the soil. The great economic consequences associated with damages to structures (particularly homes) due to these volume changes has provided the initiative to conduct research on the behavior of unsaturated, expansive soils.

2.) There is an increasing concern for the environment in developed countries. Today the geotechnical engineer is called upon to predict chemical pollutant concentrations in the ground with respect to time and space. The diffusion of a chemical is superimposed on the conductive movement within the water phase. In other words, an additional diffusive analysis has been added to the seepage of water through a soil. The hydraulic conductivity (or coefficient of permeability) of soil has often been regarded as one of the most difficult soil properties to evaluate. Now, this property is the central focus of many analyses and much research has been directed towards its quantification. Many of the processes of concern to the environment and to the water resources occur in the upper portion of the soil profile; in the vadose zone where the pore-water pressures are negative. In this zone, the hydraulic conductivity is a function of the negative pore-water pressure, resulting in nonlinear flow formulations. The groundwater table is no longer the upper boundary of concern. Rather, the ground surface geometry becomes the boundary for the problem and movements through the unsaturated zone have become of vital importance.

3.) The rapid development in the computer industry has provided the engineer with the means to handle complex geotechnical problems associated with unsaturated soils behavior. Applications software to powerful coupled, transient application software are increasingly being used on a routine basis in engineering offices. The software is often used in a parametric or sensitivity manner in order to embrace a range of possible field conditions.

ILLUSTRATIONS OF THE SCOPE OF GEOTECHNICAL PROBLEMS INVOLVING SATURATED/UNSATURATED SOIL MECHANICS

The geotechnical engineering works of man always start at the ground surface. While some projects extend well below the groundwater table, many projects remain in the zone above the groundwater table. For example, engineers place building foundations in the zone above the groundwater table for ease of construction. Whenever possible, the geotechnical engineer will attempt to do the design such that construction can remain in the unsaturated zone. In addition to shallow foundations, retaining walls and cuts in slopes are generally designed such that they remain above the water table. Over the past decades, much of the engineering design associated with soils above the water table has remained relatively empirical. At the same time, engineering designs involving soil below the water table have utilized the effective stress analyses in advancing practical designs.

Geotechnical engineers have been slow to realize the close link which exists between the area of surface hydrology and the flux boundary condition for their problems. A visualization of the hydrologic cycle quickly brings an awareness that the ground surface is a flux boundary and that water moving to the groundwater table must first pass through the unsaturated, vadose zone. Fig. 2 illustrates the artificial separation which is often placed between the hydrologic components, the vadose zone and the groundwater. In reality, the movement cycle of water is a continuous process from the atmosphere to the groundwater table. The near surface phenomena can be dynamic and extreme. Changes in the boundary conditions affect the pore-water pressures in the soil.

The building of the infrastructure for society has largely involved soils above the groundwater table. Infrastructure developments have also given rise to extensive environmental concerns (Fig. 3). Many of the environmental problems originate on or near ground surface. Contaminants must move through the unsaturated vadose zone and may eventually end up in the groundwater. An understanding of the movement of contaminants through the unsaturated, vadose zone becomes necessary when predicting the transport of chemicals from their source to their destination in the

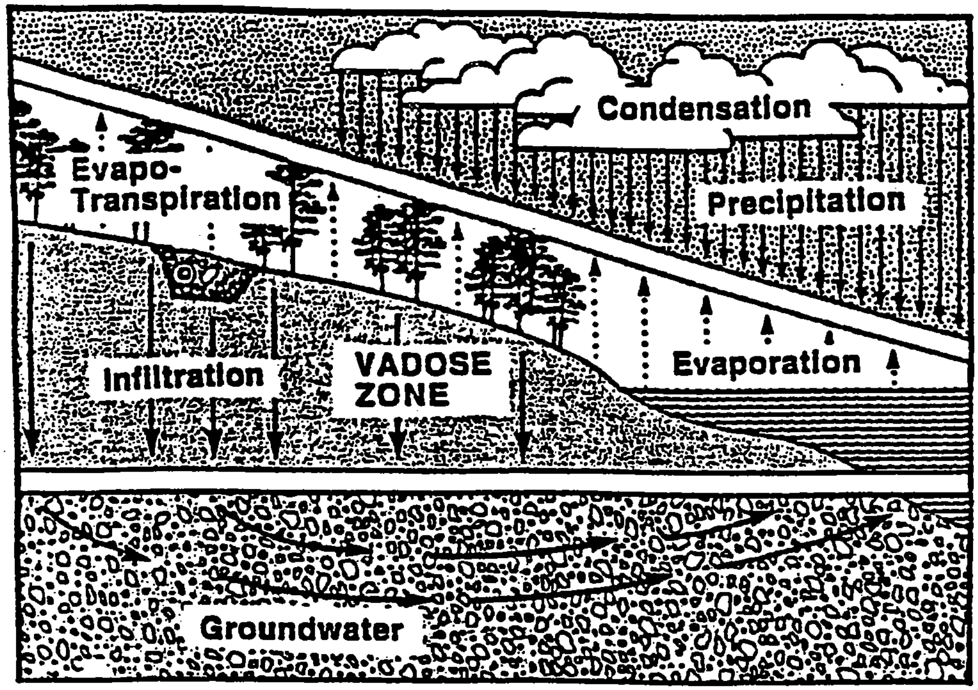
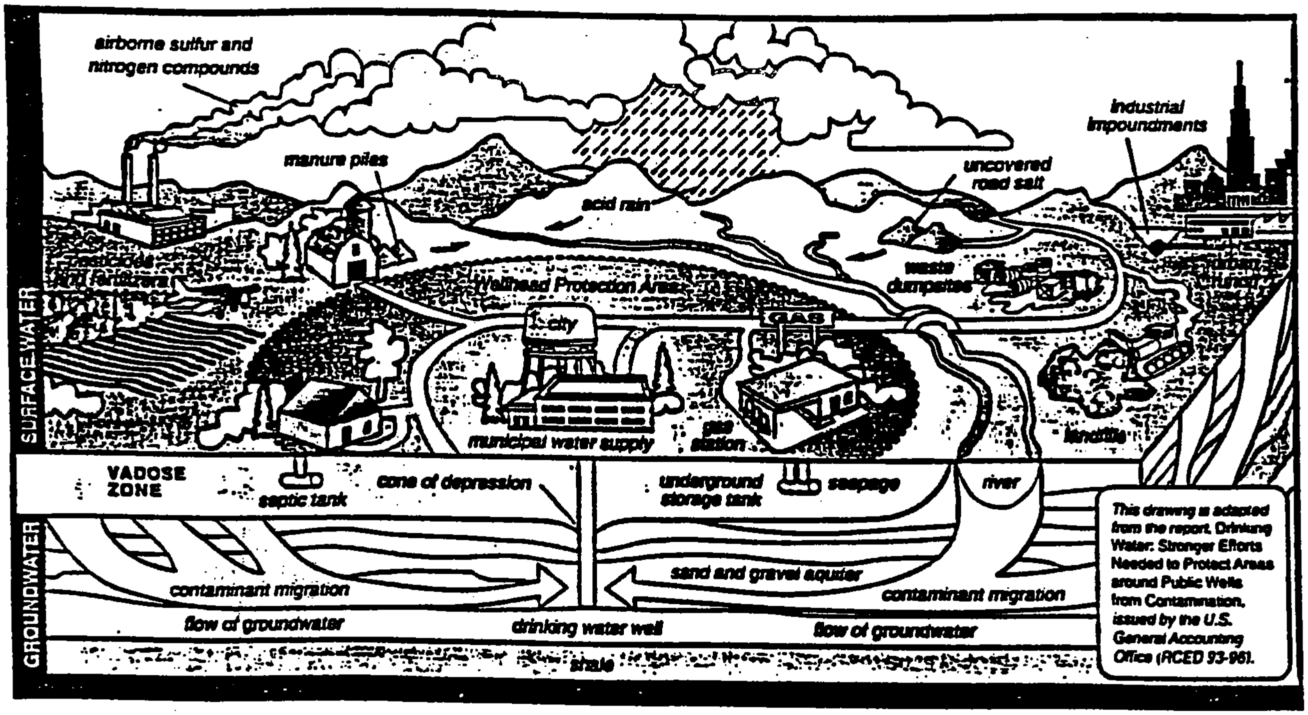


Figure 2 Schematic showing the artificially imposed boundaries between the atmosphere and the groundwater (from short course at Oklahoma University, 1994).



This drawing is adapted from the report, Drinking Water: Stronger Efforts Needed to Protect Areas around Public Wells from Contamination, issued by the U.S. General Accounting Office (RCED 93-96).

Figure 3 An illustration of the geo-environmental impact of man's activities (from Miller, 1993).

aqueous environment. Theoretical models must embrace convective flow and diffusion within the unsaturated, vadose zone.

Many geotechnical engineering projects are involved with the use of remolded, compacted soils. Dams and highway embankments are two examples of engineered structures which involve large volume of compacted soils. Problems related to compacted soils requires the application of unsaturated soil mechanics.

VISUALIZATION AIDS FOR SATURATED/UNSATURATED SOIL MECHANICS

The "geotechnical world" is presented as an ellipse with a mid-level horizontal line representing the groundwater table (Fig. 4). The situations in temperate, humid regions and in arid regions of the world can be visualized as shown in Figs. 5 and 6, respectively. In temperate, humid regions, the groundwater table may be close to the ground surface (Fig. 5). In arid regions, the groundwater table can be very deep (Fig. 6).

Below the water table, the pore-water pressures will be positive and the soils will, in general be saturated. Above the water table, the pore-water pressures will, in general, be negative. Immediately above the water table is a zone called the capillary fringe where the degree of saturation approaches 100 percent. This zone may range from less than one metre to approximately 10 metres in thickness, depending upon the soil type. The entire zone above the water table is called the vadose zone.

The negative pore-water pressures above the water table are generally referenced to the pore-air pressure (i.e., $(u_a - u_w)$). The difference between the pore-air pressure and the pore-water pressure is called the matric suction. The term originated in Soil Science and was later shown to be one of two stress state variables required to describe the behavior of an unsaturated soil.

Definition of Soil Suction

Suction in an unsaturated soil is made up of two components; namely, matric suction and osmotic suction. The sum of the two components is called total suction. Matric suction is defined as the difference between the pore-air pressure, u_a, and the pore-water pressure, u_w (i.e., matric suction = $u_a - u_w$).

The osmotic suction is a function of the amount of dissolved salts in the pore fluid, and is written in terms of a pressure.

The matric suction is of primary interest because it is the stress state variable which is strongly influenced by environmental changes.

The terminology, saturated soil mechanics and unsaturated soil mechanics, would suggest that it is the degree of saturation which provides the distinction between these two categories (Fig. 7). While this is true, it is difficult to measure the degree of saturation and to use the degree of saturation in the analysis of unsaturated soil behavior. It is rather the state of stress in the water phase which has become the primary indicator for classification purposes. Any soil with a negative pore-water pressure is considered to be part of unsaturated soil mechanics. It is recognized that soils with negative pore-water pressures can be saturated or contain air bubbles in an occluded form. Fig. 8 shows a soil classification based on the continuity of the air and water phases.

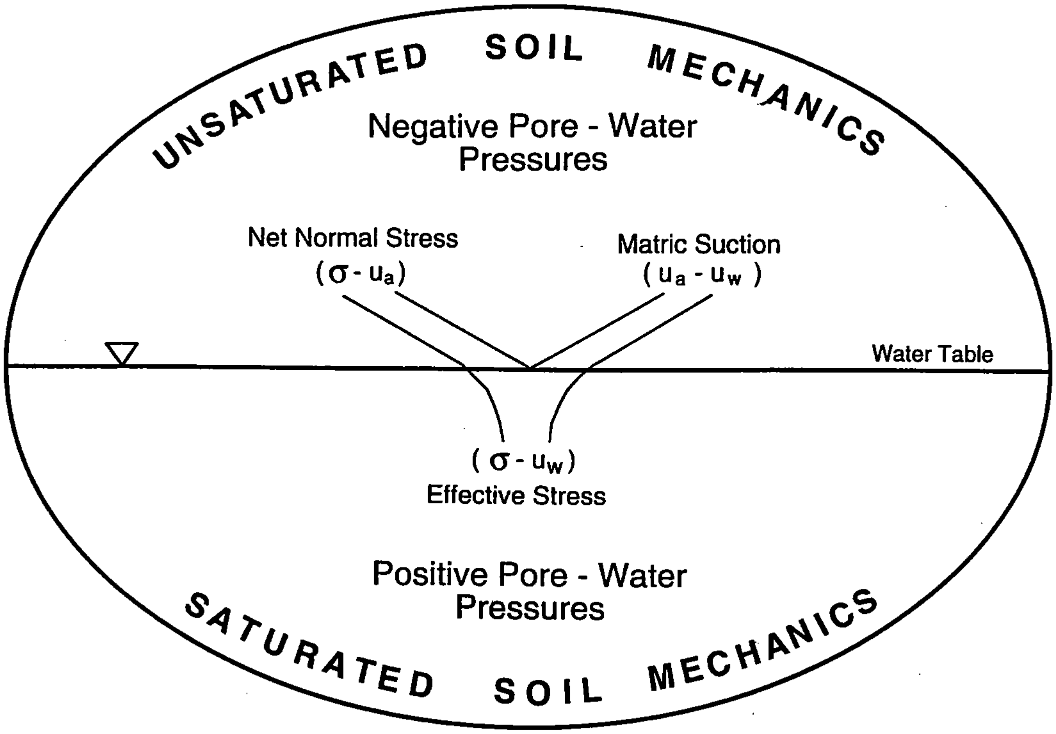


Figure 4 A visualization aid for the generalized world of soil mechanics.

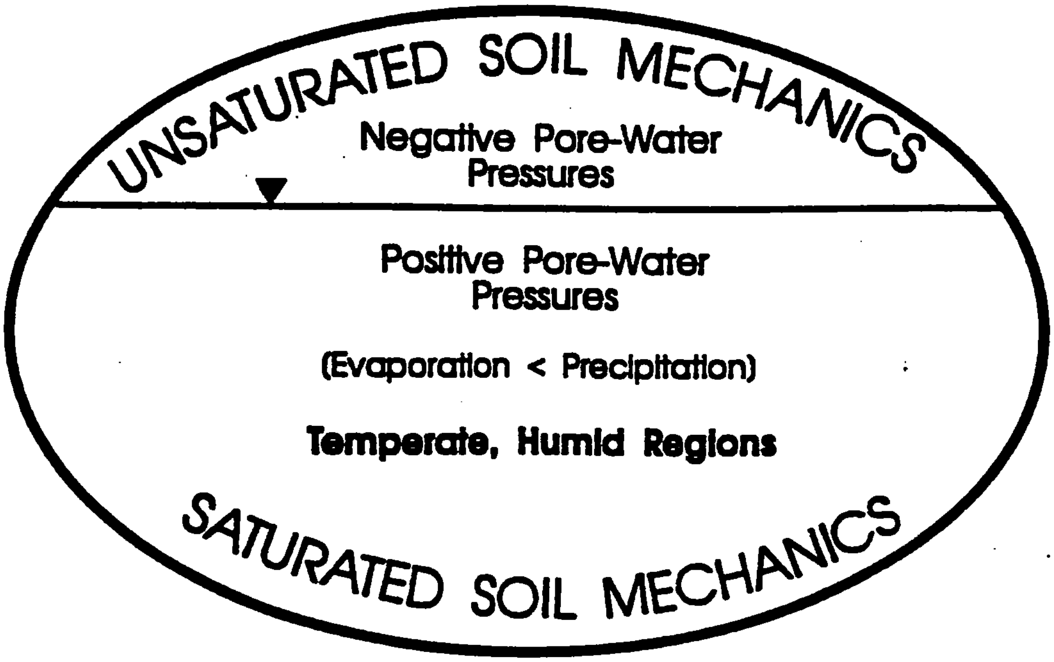


Figure 5 A visualization aid for the temperate, humid regions.

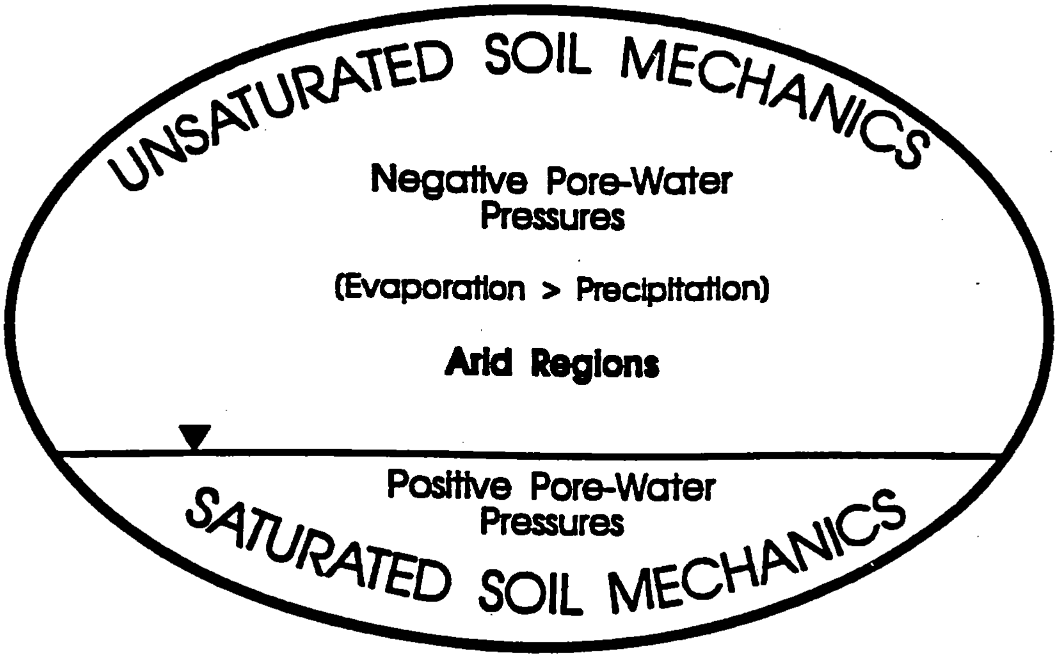


Figure 6 A visualization aid for the arid regions.

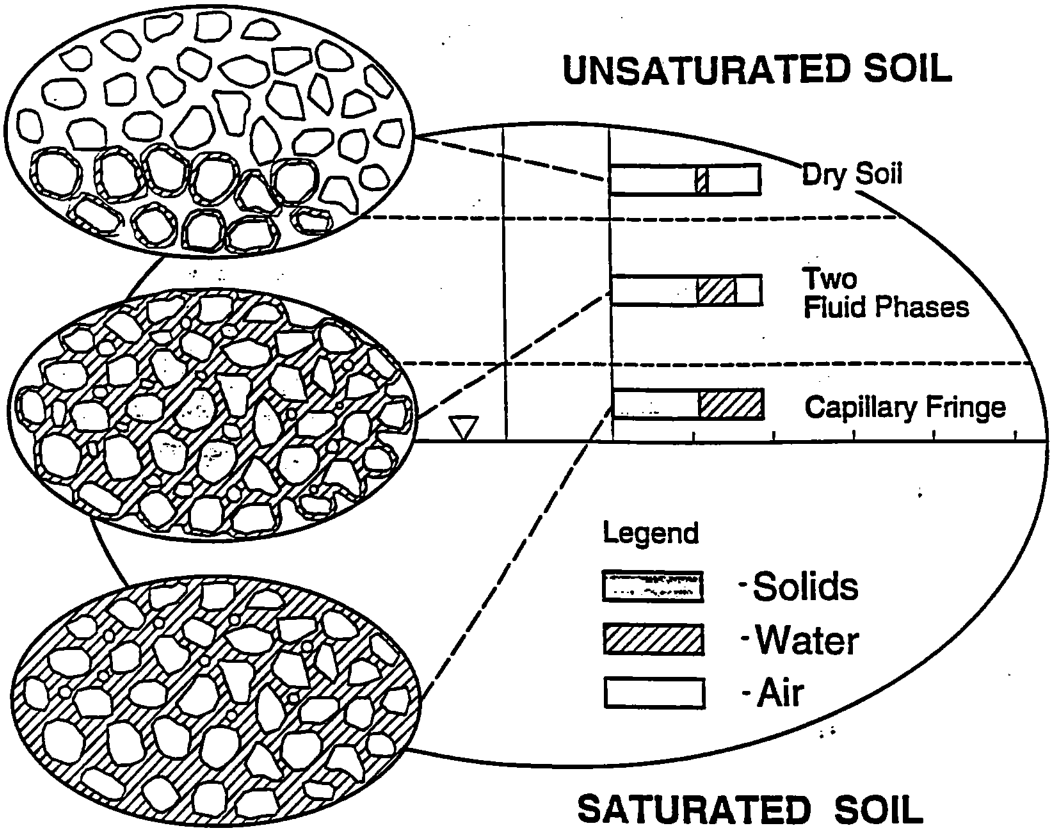


Figure 7 Categorization of soil above the water table based on the variation in degree of saturation.

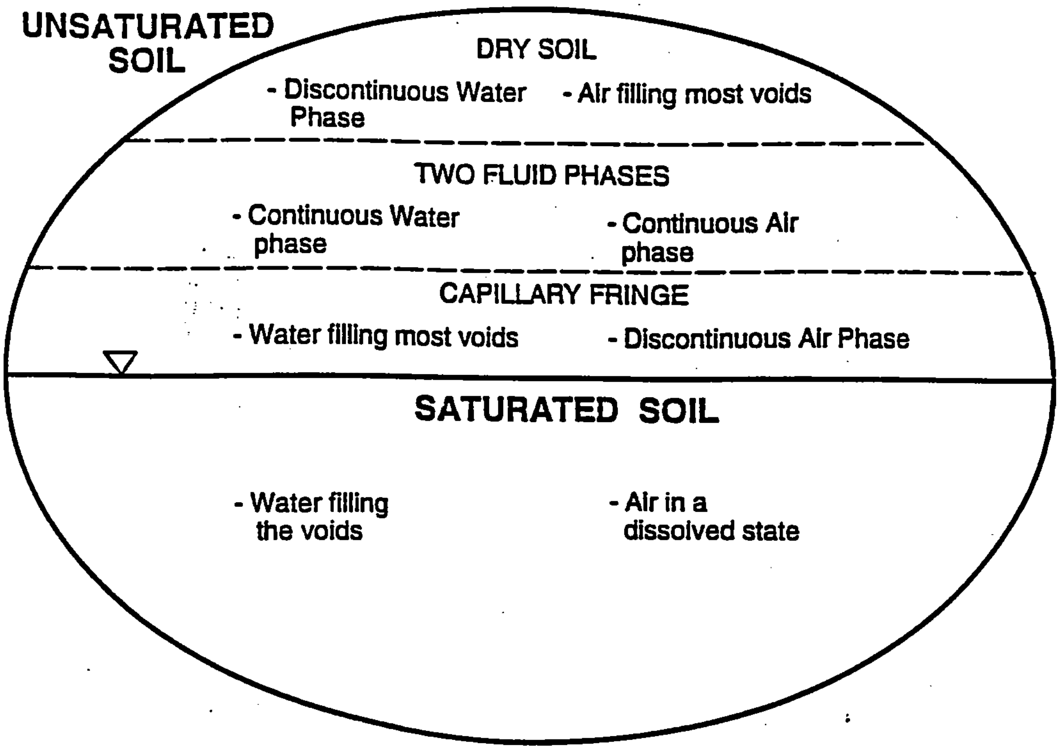


Figure 8 A visualization of saturated/unsaturated soil mechanics based on the nature of the fluid phases.

The physics and engineering principles involved with dry soils are essentially the same as those involved with saturated soils. The difference between a completely dry and a completely saturated soil is related to the compressibility of the pore fluid. The water in a saturated soil is essentially incompressible. The water becomes compressible as air bubbles appear in the water. Most attention in research has been given to the case where air and water are continuous throughout the voids. It appears to be the case most relevant to engineering practice.

CATEGORIZATION OF SATURATED/UNSATURATED SOIL MECHANICS

Saturated/unsaturated soil mechanics can be categorized in a manner consistent with the traditional classic areas of soil mechanics. There are three main categories; namely, seepage, shear strength and volume change. This subdivision, along with the basic equations associated with each category, are shown in Fig. 9. This categorization shows that the same types of problems are of interest for both saturated and unsaturated soils. It also shows that the behavior of an unsaturated soil is a function of the stress states. The stress state categorization provides the basis for a science which is transferable the world over. An unsaturated soil may either increase or decrease in overall volume when subjected to wetting (i.e., a decrease in matric suction). If the volume increases upon wetting, the soil has a swelling nature and if the volume decreases upon wetting, the soil has a collapsing nature. The difference in behavior is associated with the structure of the soil. The swelling soil has a "stable-structure" and the collapsing soil has a "meta-stable-structure". This categorization also applies to both remolded (i.e., compacted) and natural

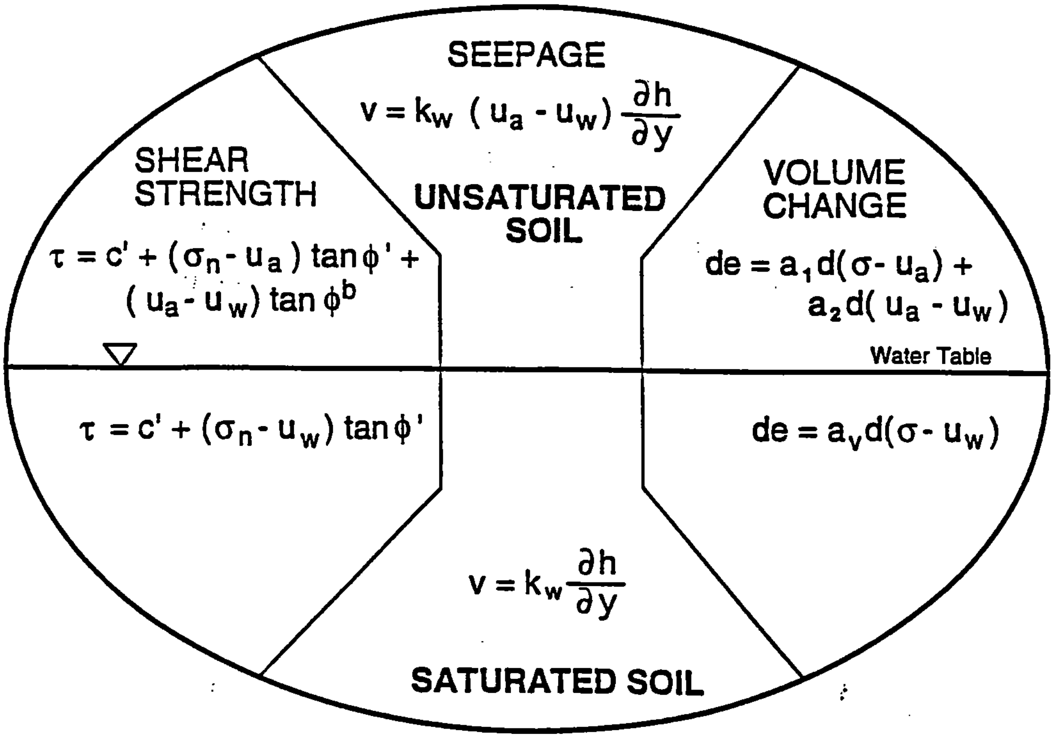


Figure 9 Categorization of soil mechanics based on the type of engineering problem.

soils. The awareness of these types of behavior is important since the constitutive relationships (i.e., volume change behavior in particular), are different for each case. The case for swelling of a stable-structured soil is illustrated in Fig. 10. The soil suction is shown to generally increase as ground surface is approached. Likewise, the potential for swelling increases exponentially as the ground surface is approached. This is true of soils with high plasticity.

The character of the nonlinear coefficient of permeability function is illustrated in Fig. 11. Once the air entry value of the soil is exceeded, the coefficient of permeability decreases from its saturated value. The decrease in coefficient of permeability bears a linear log-log correspondence to soil suction. Various mathematical forms have been proposed for the coefficient of permeability function. Shown on Fig. 11 is the mathematical form known as Gardner's equation (Gardner, 1958).

It is apparent from Fig. 9 that the basic constitutive relations for each category of problem are more complex for unsaturated soils than for saturated soils. At the same time, each of the equations specializes from the unsaturated case to the saturated case as the matric suction term goes to zero. A second soil parameter, ϕ^b, is required when describing the shear strength of an unsaturated soil. The coefficient of permeability becomes a nonlinear function for the analysis of seepage problems. Two independent coefficients of compressibility are required to define void ratio changes for an unsaturated soil. As well, changes in water content must be predicted using an independent constitutive relation when analyzing unsaturated volume-mass changes.

In addition to the above, soils can also be categorized genetically. The geologic genesis of a deposit has certain characteristics which indicate the nature of engineering properties likely to be encountered. Some of the common genetic categories can be listed as follows: Lacustrine soils, Bedrock soils, Aeolian soils, Alluvial soils, Residual soils and others (Fig. 12). From the standpoint of a theoretical framework, this categorization has little to offer in describing the

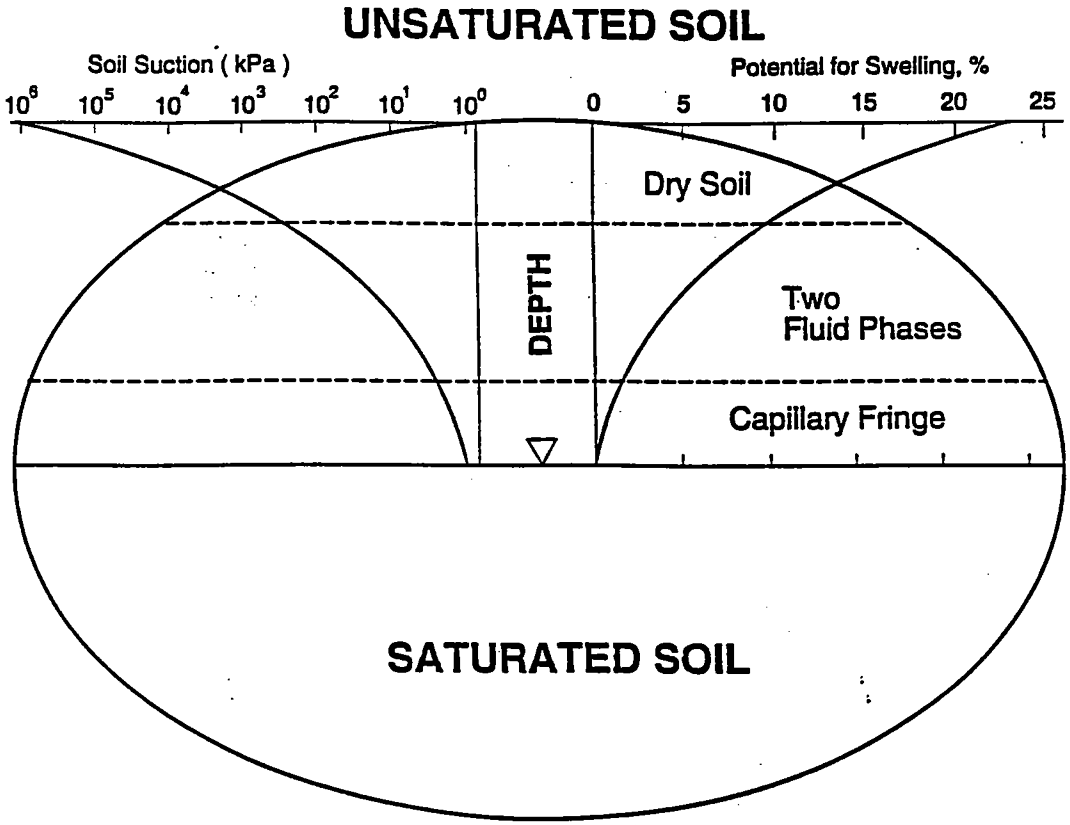


Figure 10 An illustration of the potential for swelling versus depth and soil suction.

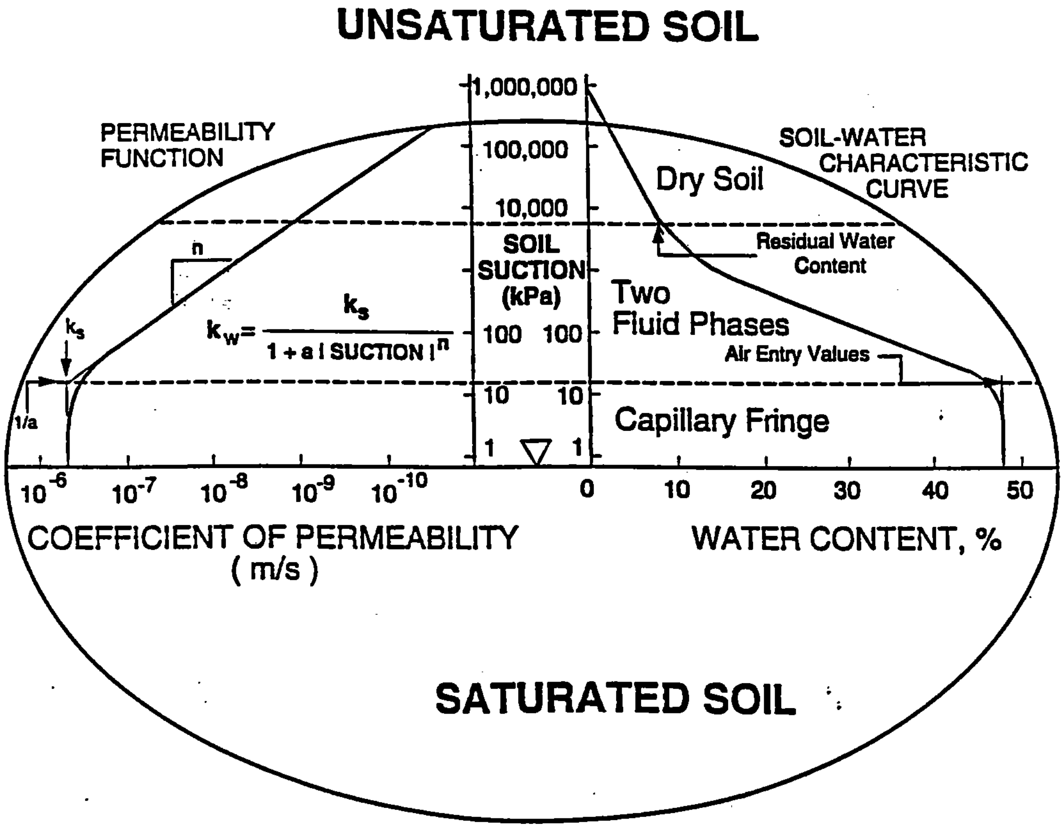


Figure 11 Visualization of the coefficient of permeability function in the unsaturated soil zone.

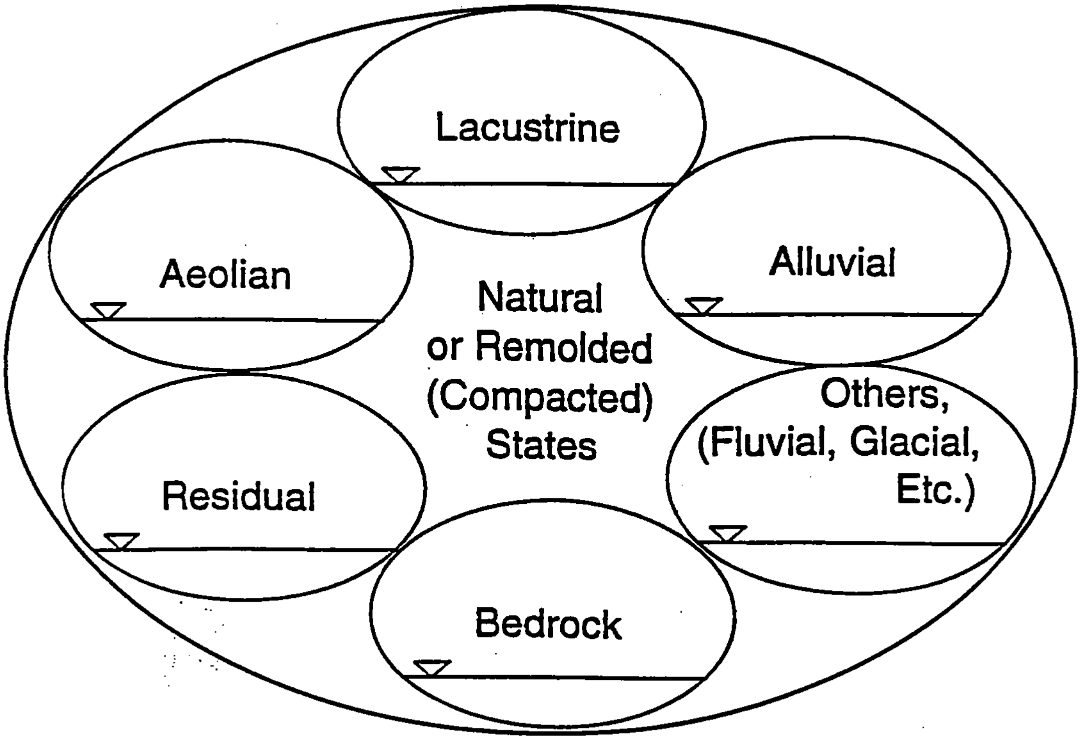


Figure 12 Categorization of soil mechanics based on geologic origins.

behavior of soils. In other words, essentially the same theories can be shown to apply for all types of soil.

There are also textural and plasticity categories of soils (i.e., sands, silts and clays). Regardless of the various soil categorizations, similar engineering problems can be encountered.

CLIMATE AND THE VADOSE ZONE

The location of the groundwater table (i.e., separation between negative and positive pore-water pressures) is strongly influenced by the long term climatic conditions in a region (Figs. 5 and 6). If the region is arid or semi-arid, the groundwater table is slowly lowered with time (i.e., geologic time scale) (Fig. 6). If the climate is temperate or humid, the groundwater table may remain quite close to the ground surface (Fig. 5). It is the difference between the downward flux (i.e., precipitation) and the upward flux (i.e., evaporation and evapotranspiration) on a long term basis, which determines the location of the groundwater table (Fig. 13).

The portion of the soil profile above the groundwater table, called the vadose zone, can be readily subdivided into two portions. The portion immediately above the water table, called the capillary fringe, remains saturated even though the pore-water pressures are negative. The portion above the capillary fringe is unsaturated.

Regardless of the degree of saturation of the soil, the pore-water pressure profile in Fig. 13 will come to equilibrium at a hydrostatic condition when there is zero flux from the ground surface. If moisture is extracted from the ground surface (e.g., evaporation), the pore-water pressure profile will be drawn to the left. If moisture enters at the groundwater surface (e.g., infiltration), the pore-water profile will be drawn to the right.

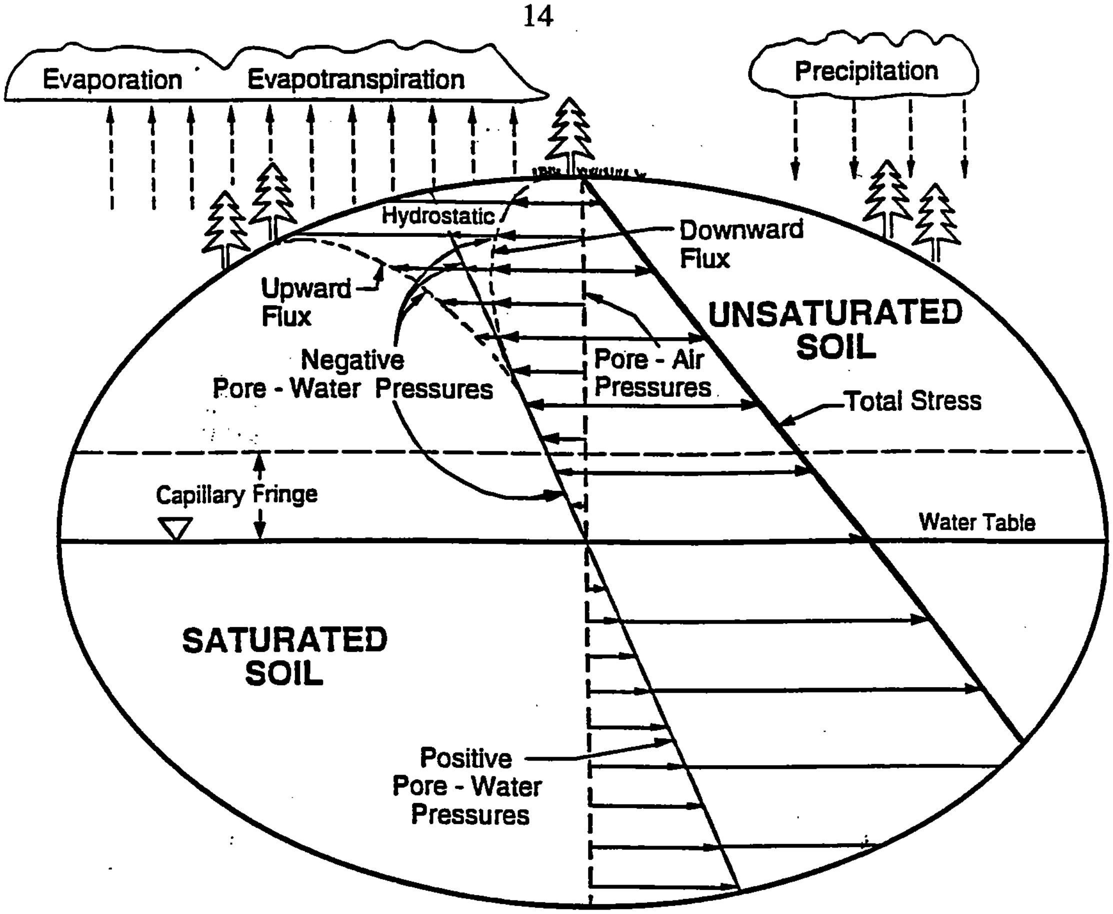


Figure 13 A visualization of soil mechanics showing the role of the surface flux boundary condition.

The precipitation conditions at a site are often known from past records and are available for design purposes. The evaporative flux, on the other hand, must be computed through the use of one of several models. Some of the most approximate estimates for geotechnical problems have been associated with the assessment of evaporative flux. Only recently have engineers began to include the role of soil suction in computing the rate of evaporation from the ground surface (Wilson, 1990). Figure 14 shows that the rate of release of water to the atmosphere, from a sand, silt or clay, is related to the suction in the soil. The computation of the evaporative flux has become an area of research which is of great value to engineers dealing with geo-environmental problems.

There are many complexities associated with the vadose zone because of its fissured and fractured nature. The tendency in geotechnical engineering has been to avoid the analysis of this zone, if possible. However, in many cases it is an understanding of this zone which holds the key to the performance of an engineered structure.

Historically, classical seepage problems involved saturated soils where the boundary conditions consisted of either a designated head or zero flux. However, the real world for the engineer often involved a ground surface, where there may be a positive or negative flux condition. Geo-environmental type problems has done much to force the engineer to give consideration of saturated/unsaturated seepage analyses with flux boundary conditions. The improved computing capability available to the engineer has assisted in accommodating these changes.

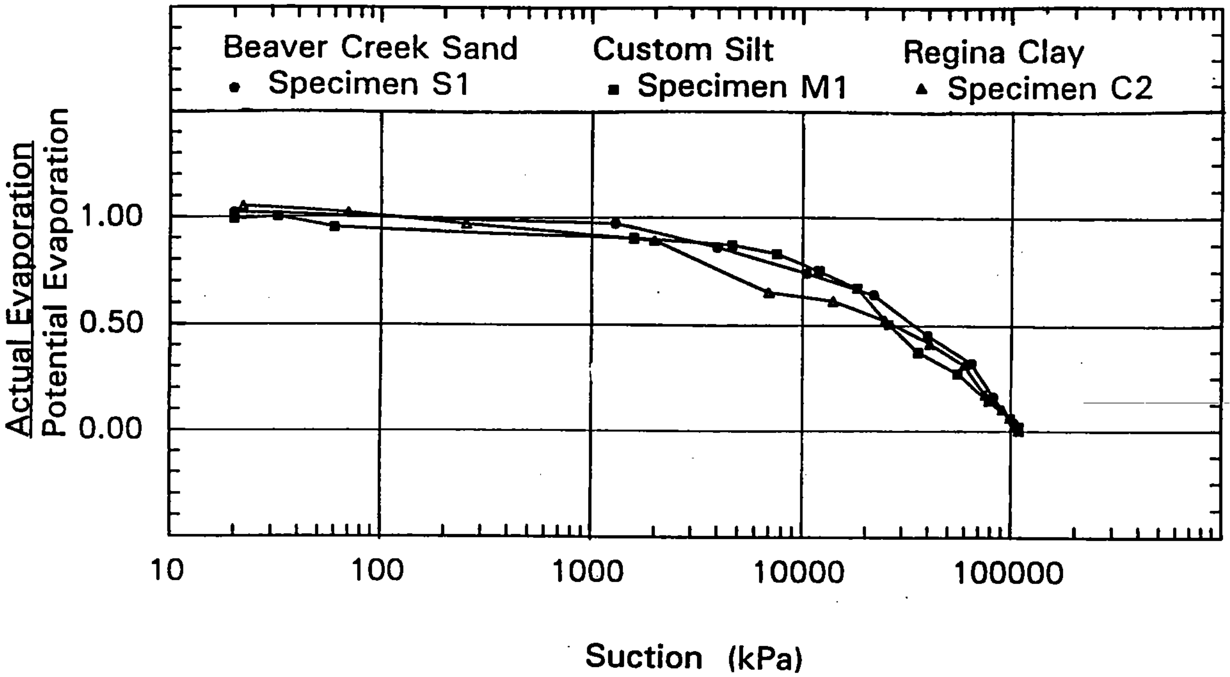


Figure 14 Relationship between the ratio of actual evaporation to potential evaporation, and soil suction for three soils (from Wilson, 1991).

Most of the manmade structures are sited on the surface of the earth (Fig. 15) and as such will have an environmental flux boundary condition. Consider, for example, a highway where the soil in the embankment and subgrade have an initial set of conditions or stress states. These conditions will change with time primarily because of environmental (or surface moisture flux) changes. The foundations for light structures are likewise generally placed well above the groundwater table where the pore-water pressures are negative. In fact, most of the light engineered structures of the world are placed within the vadose zone.

One of the characteristics of the upper portion of the vadose zone is its ability to slowly release water vapor to the atmosphere at a rate dependent upon the permeability of the intact portions of soil. At the same time, downward flow of water can occur through the fissures under a gradient of unity. There appears to be no impedance to the inflow of water until the soil swells and the mass becomes intact, or until the fissures and cracks are filled with water.

A common misconception is that water can always enter the soil at the ground surface. However, if the soil is intact, the maximum flux of water at the ground surface is equal to the saturated coefficient of permeability of the soil. This value may be extremely low. If the ground surface is sloping, the surface layer can become saturated and have a higher coefficient of permeability than the underlying soil. As a result, water runs down the top layers of soil on the slope, and may not enter the underlying soil.

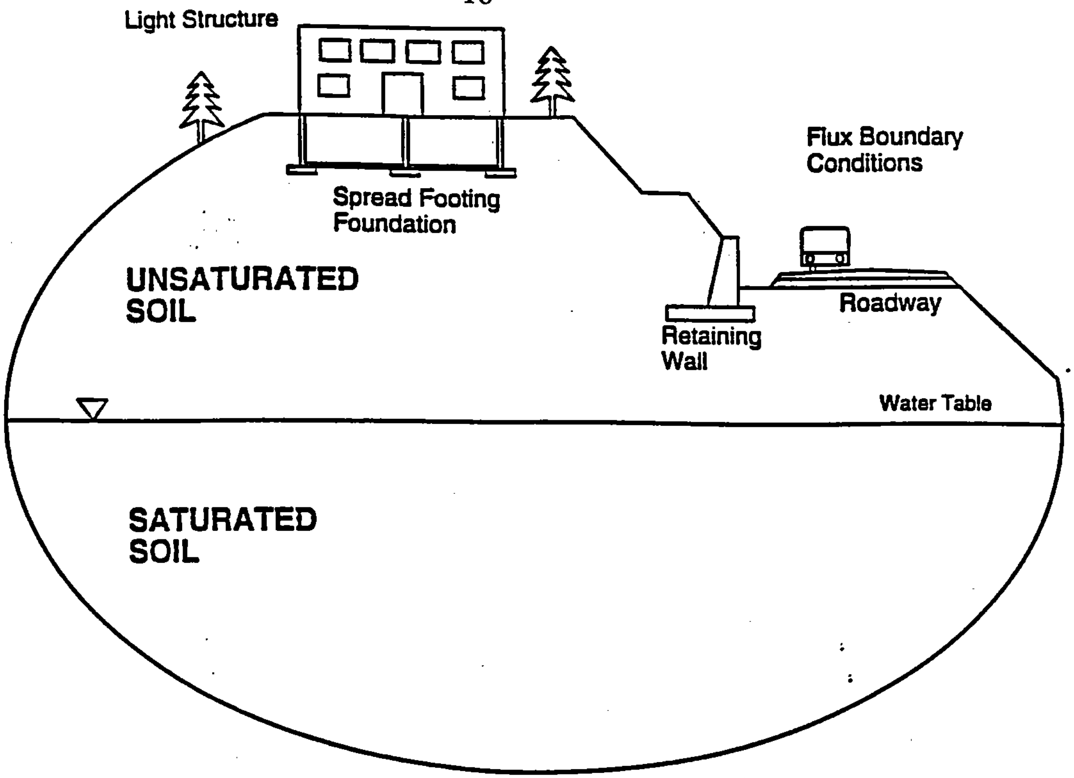


Figure 15 Examples of engineered structures commonly placed above the water table.

STRESS STATE OF A SOIL

The stress state of a saturated soil is completely defined by a set of effective stress variables,

$$[1] \qquad [\sigma'] = \begin{bmatrix} (\sigma_x - u_w) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_w) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_w) \end{bmatrix}$$

where

$\sigma_X, \sigma_y, \sigma_Z$ = total stresses in x, y, z directions, respectively

u_w = pore-water pressure.

Using a similar stress state approach, it was established that the stress state of an unsaturated soil is defined by two sets of independent stress state variables (Fredlund and Morgenstern, 1977). There are three sets of possible stress state variables, of which only two of the three sets are independent.

Lytton (1978) noted that it was possible to form independent stress tensors from the proposed stress state variables. The stress tensors associated with the three sets of independent stress state variables are:

[2a] Stress state set No. 1 :
$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix}$$

[2b] Stress state set No. 2 :
$$\begin{bmatrix} (\sigma_x - u_w) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_w) & \tau_{xy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_w) \end{bmatrix}$$

and

[2c] Stress state set No. 3 :
$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix}$$

In the special case of a saturated soil, the air pressure, u_a, is equal to the water pressure, u_w, and the three stress tensors reduce to one single stress tensor (i.e., Eq. [1]).

The stress state variables most often used for unsaturated soils are the following two stress state variables:

[3a]
$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix}$$

and

[3b]
$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix}$$

The stress state at a point in an unsaturated soil element, in terms of the above two sets of stress state variables are shown in Fig. 16.

STRESS INVARIANTS

The three principal stresses on three mutually orthogonal planes for a generalized soil element are $(\sigma_1 - u_a)$, $(\sigma_2 - u_a)$ and $(\sigma_3 - u_a)$ respectively. In addition, the matric suction, $(u_a - u_w)$, also acts equally on all three planes.

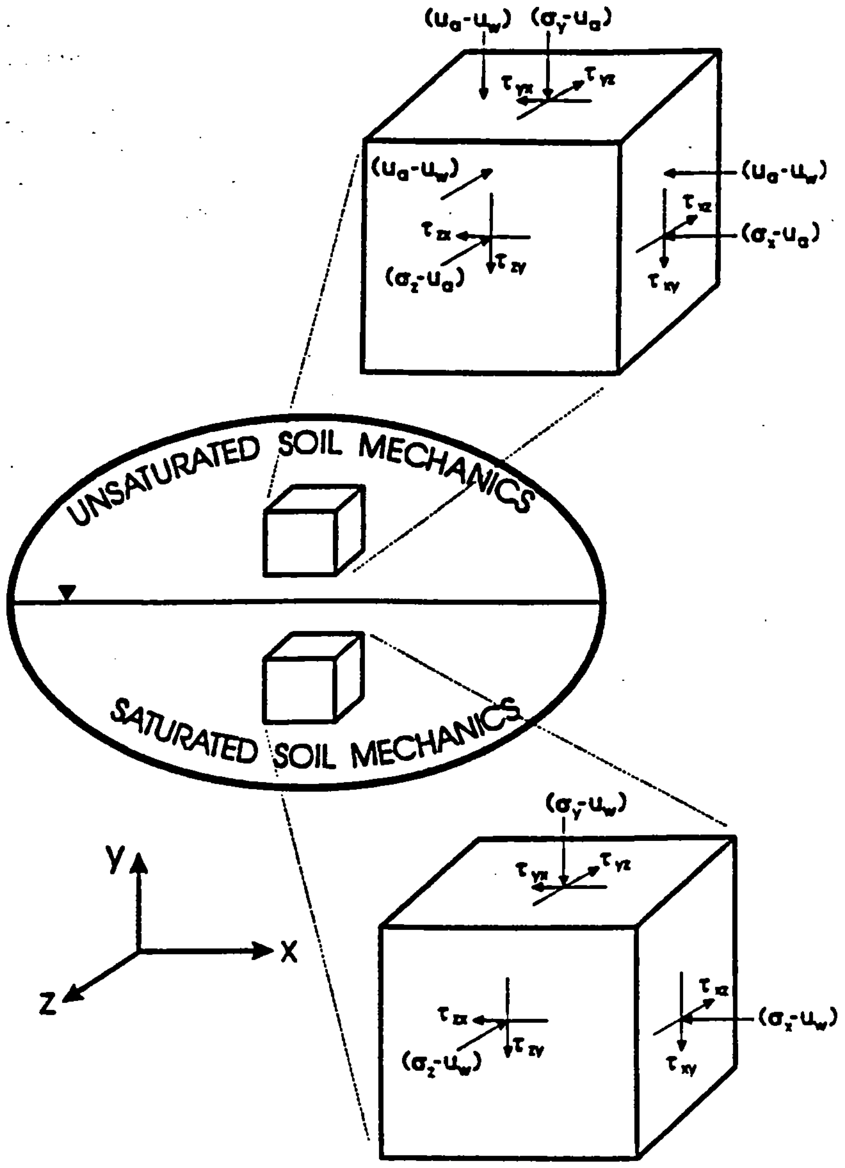


Figure 16 Elements showing the stress state at a point in a saturated soil and in an unsaturated soil.

There are three stress invariants that can be derived from each of the two independent stress tensors corresponding to the stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, for an unsaturated soil. The first stress invariants of the stress tensors associated with the stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$ are:

[4a] $$I_{11} = \sigma_1 + \sigma_2 + \sigma_3 - 3u_a$$

and

[4b] $$I_{12} = 3(u_a - u_w)$$

where

I_{11} = first stress invariant of the the first tensor and

I_{12} = first stress invariant of the second tensor.

The second stress invariants of the same set of stress tensors are:

[5a] $$I_{21} = (\sigma_1 - u_a)(\sigma_2 - u_a) + (\sigma_2 - u_a)(\sigma_3 - u_a) + (\sigma_3 - u_a)(\sigma_1 - u_a)$$

and

[5b] $$I_{22} = 3(u_a - u_w)^2$$

where :

I_{21} = second stress invariant of the the first tensor

I_{22} = second stress invariant of the second tensor.

The third stress invariants of the same set of stress tensors are:

[6a] $$I_{31} = (\sigma_1 - u_a)(\sigma_2 - u_a)(\sigma_3 - u_a)$$

and

[6b] $$I_{22} = (u_a - u_w)^3$$

where :

I_{31} = third stress invariant of the first tensor

I_{32} = third stress invariant of the second tensor.

The stress invariants of the second tensor, I_{12}, I_{22} and I_{32} are related. Therefore, only one stress invariant is required to represent the second tensor associated with the stress state variable $(u_a - u_w)$.

A total of four stress invariants (i.e., three for the 1st stress tensor corresponding to the state variable $(\sigma - u_a)$, and one for the stress tensor corresponding to the state variable, $(u_a - u_w)$) are therefore required to characterize the stress state of a generalized soil element. When the soil is saturated, u_a is equal to u_w, and the stress invariants corresponding to the state variable, $(u_a - u_w)$, of the second tensor is zero. The remaining three stress invariants corresponding the state variable, $(\sigma - u_a)$, revert to the same form as that require to characterize the stress state of a saturated soil.

STRESS POINT DESIGNATIONS

Some researchers have found that it is more convenient to deal with stress points, defined by *p* and *q*, rather than with principal stresses, σ_1, σ_2 and σ_3, or with the stress invariants. Two different systems of stress point designations have been defined. These are the MIT designation (Lambe and Whitman, 1969) and the Critical State Soil Mechanics designation (Wheeler and Sivakumar, 1992). Similar designations for the stress points, *p*, *q* and *r*, for a generalized soil element can also be written (Fredlund and Rahardjo, 1993).

MIT Designation for Stress Point Parameters

The MIT designation for stress point, *p* and *q*, for a *saturated* soil are defined as:

[7a] $$p = \frac{\sigma_1 + \sigma_3}{2} - u_w$$

and

[7b] $$q = \frac{\sigma_1 - \sigma_3}{2}.$$

In an *unsaturated* soil, the corresponding stress point are denoted by *p*, *q* and *r*, have been defined (Fredlund and Rahardjo, 1993):

[8a] $$p = \frac{(\sigma_1 - u_a) + (\sigma_3 - u_a)}{2}$$

or $$p = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - u_a$$

[8b] $$q = \frac{\sigma_1 - \sigma_3}{2}$$

and

[8c] $$r = (u_a - u_w).$$

The stress points, *p* and *q*, using the MIT designation were originally obtained by considering only the state of stress in the plane that contains the major and the minor principal stresses, σ_1 and σ_3. It should be noted that the p and q parameters so defined should appropriately be applied

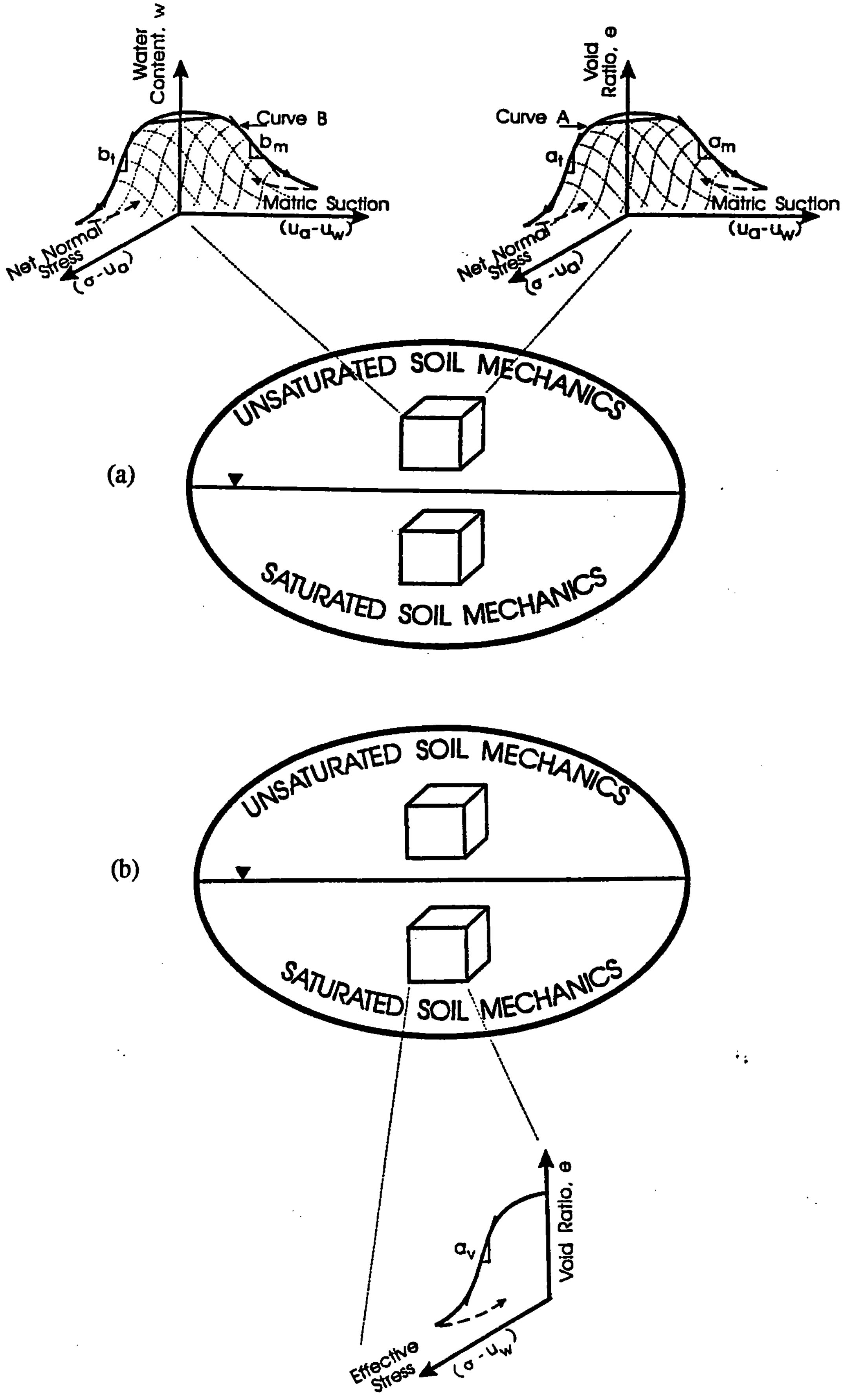


Fig. 17 Void ratio and water content constitutive surfaces for a.) unsaturated soil, and for b.) saturated soil.

only to stresses on a failure plane. These parameters would not be totally appropriate for describing general stress state conditions prior to failure.

Critical State Soil Mechanics Designation for Stress Point Parameters

The Critical State Soil Mechanics designation (Wheeler, 1992), for a *saturated* soil, defined in terms of the stress points, p and q, for triaxial conditions are as follows:

[9a] $$p = \frac{\sigma_1 + 2\sigma_3}{3} - u_w$$

and

[9b] $$q = \sigma_1 - \sigma_3$$

The corresponding stress points, p, q and r for an *unsaturated* soil are:

[10a] $$p = \left(\frac{\sigma_1 + 2\sigma_3}{3}\right) - u_a$$

[10b] $$q = \sigma_1 - \sigma_3$$

and

[10c] $$r = (u_a - u_w)$$

For general three-dimensional stress state conditions in a *saturated* soil, the stress points denoted by p and q can be written as:

[11a] $$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} - u_w$$

and

[11b] $$q = \sqrt{\frac{1}{2}\left[(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2\right]}$$

The corresponding stress point, p, q and r, for the general stress state in an *unsaturated* soil are respectively:

[12a] $$p = \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right) - u_a,$$

[12b] $$q=\sqrt{\frac{1}{2}\left[(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+(\sigma_1-\sigma_2)^2\right]}$$

and

[12c] $$r=(u_a-u_w)$$

The stress points according to the Critical State Soil Mechanics designation are applicable under working load conditions as well as under failure conditions.

Plotting of Stress Points for an Unsaturated Soil

Since there are three parameters, p, q and r, associated with each stress point for an unsaturated soil, the stress states for an unsaturated soil are plotted in three dimensions with q as the ordinate and p and r as the abscissas.

THEORETICAL FORMULATIONS FOR SATURATED/UNSATURATED SOIL MECHANICS

The formulations for volume change, shear strength and flow in a soil element are briefly summarized in this section.

Volume Change Moduli

The volume change constitutive behavior of an unsaturated soil can be written in several possible forms. Only the classical soil mechanics form involving void ratio, e, water content, w and/or degree of saturation, S, is shown in this paper. The independent stress state variables can be used to formulate the constitutive relations for an unsaturated soil element. The volume change constitutive equation for isotropic loading, (σ_c-u_a), written in terms of void ratio, *e*, is,

[13] $$de=a_t\,d(\sigma_c-u_a)+a_m\,d(u_a-u_w)$$

where:

$$a_t=\frac{\partial e}{\partial(\sigma-u_a)}$$

and

$$a_m=\frac{\partial e}{\partial(u_a-u_w)}$$

The void ratio change can be independent of the water content change for an unsaturated soil element. For a complete volume-mass characterization, a second constitutive relationship is required. The water content constitutive relationship is generally used and can be written as follows for the case of isotropic loading,

[14] $$dw = b_t\, d(\sigma_c - u_a) + b_m\, d(u_a - u_w)$$

where :

$$b_t = \frac{\partial w}{\partial(\sigma - u_a)}$$

and

$$b_m = \frac{\partial w}{\partial(u_a - u_w)}$$

Similar sets of constitutive equations can also be written for non-isotropic loading conditions.

A similar approach can be applied in the critical state framework as proposed by Wheeler and Sivakumar (1992), which use three parameters, p, q, r, along with two additional state parameters (i.e., water content, w, and specific volume, v). The specific volume, v, and void ratio, e, are related as follows:

[15] $$v = 1 + e$$

The void ratio and water content constitutive surfaces corresponding to Eqs. 13 and 14 can be represented graphically as shown in Fig. 17, where the water content, w, or void ratio, e, are plotted against the ordinates, $(\sigma - u_a)$ and $(u_a - u_w)$.

The void ratio constitutive surfaces for the loading and unloading of a stable-structured soil are shown in Fig. 18. The void ratio constitutive surfaces of a meta-stable-structured soil are shown in Fig. 19.

Shear Strength

The shear strength equation for an unsaturated soil can be formulated as a linear combination of the stress state variables, (i.e., $(\sigma_n - u_a)$ and $(u_a - u_w)$):

[16] $$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

where c' = effective cohesion

ϕ' = effective angle of internal friction

ϕ^b = friction angle associated with the matric suction stress state variable, $(u_a - u_w)$

$(\sigma_n - u_a)$ = net normal stress.

Three-dimensional representations of Eq. 16 are presented in Fig. 20. It has been found that the shear strength relationship involving suction can be either linear or nonlinear. In general it is possible to linearize the nonlinear shear strength versus suction relationship over a selected range of soil suction.

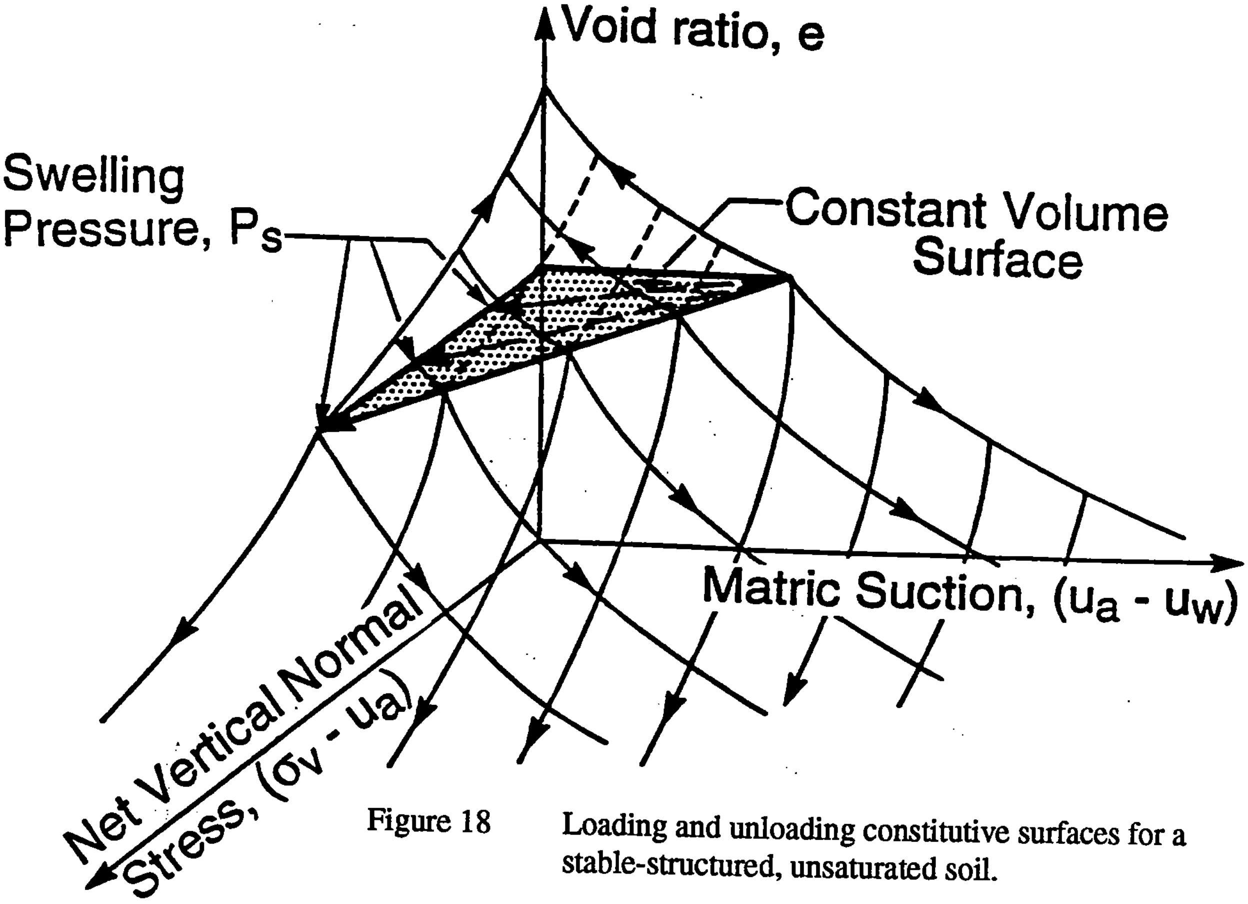


Figure 18 Loading and unloading constitutive surfaces for a stable-structured, unsaturated soil.

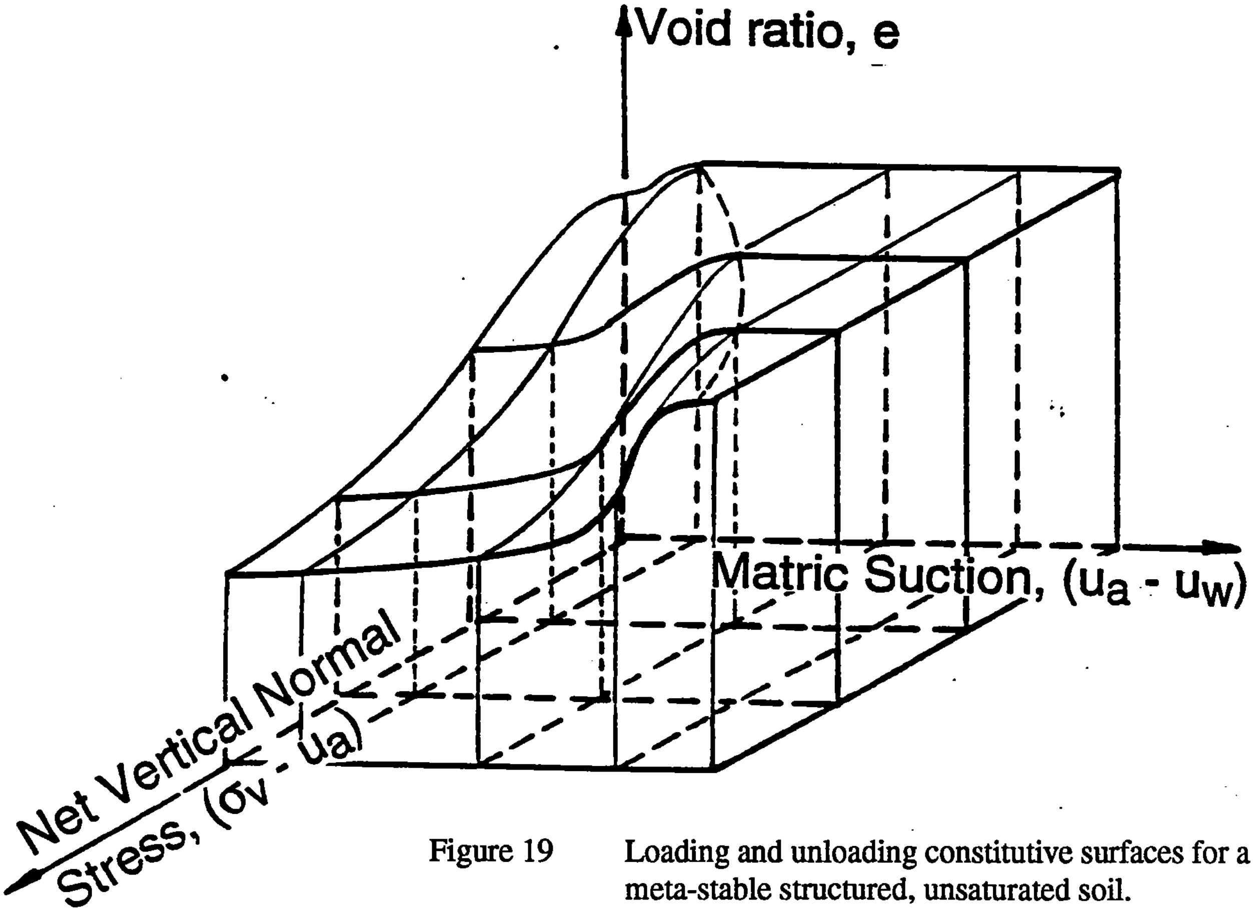


Figure 19 Loading and unloading constitutive surfaces for a meta-stable structured, unsaturated soil.

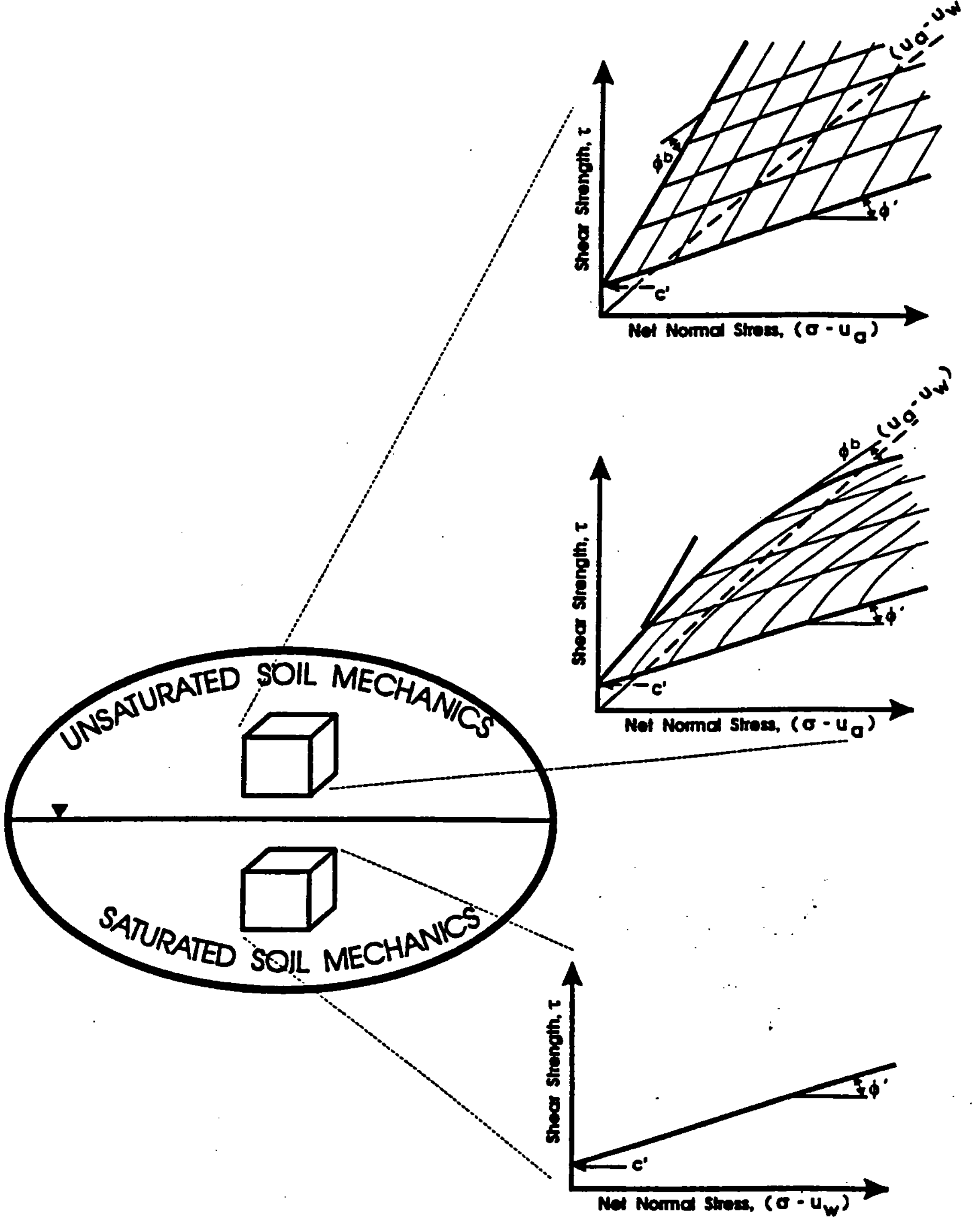


Figure 20 Mohr-Coulomb shear strength envelopes for a.) unsaturated soil, and for b.) saturated soil.

Flow

Darcy's flow law which states that the velocity of flow is proportional to the hydraulic head gradient, is applicable to both saturated and unsaturated soil media. For a saturated or unsaturated soil, Darcy's flow equation can be written as follows:

$$v_w = -k_w(u_a - u_w)\frac{\partial h}{\partial y} \tag{17}$$

where :

v_w = flow rate of water

$k_w(u_a - u_w)$ = coefficient of permeability with respect to the water phase, as a function of matric suction, $(u_a - u_w)$.

$\partial h_w / \partial y$ = hydraulic head gradient in the y-direction.

In a saturated soil, the coefficient of permeability (or hydraulic conductivity) is a constant. The constant coefficient of permeability of saturated soil is due to constant value for the water content of the saturated soil. The water content is related to the porosity available for water flow. The water content, on the other hand, decreases as a soil desaturates. The dependence of the coefficient of permeability on the water content is based on the assumption that water can only flow through the wetted portion of the soil as shown in Fig. 21. An integration along the soil-water characteristic curve provides a measure of the quantity of water in the soil. The permeability function represented by Gardner's equation (1958) have been found to fit the computed values obtained from the soil-water characteristic curve (Fig. 22).

RELATIONSHIP BETWEEN SOIL-WATER CHARACTERISTIC CURVES AND BEHAVIOR OF UNSATURATED SOILS

The soil-water characteristic curve of a soil has been shown to be closely related to unsaturated soil functions (Fredlund, 1993). Laboratory tests have indicated that there is unique relationship between the behavior of a soil and the soil-water characteristic curve of the soil. The soil-water characteristic curve defines the relationship between the volume of water in the soil and the matric suction of a soil.

Figure 23 shows a typical plot of a soil-water characteristic curve for a silty soil. The key features of a soil-water characteristic curve are identified in Fig. 23. The soil-water characteristic curve is hysteretic under drying and wetting conditions. It is usually sufficient to only consider the desorption curve. The air entry value is the matric suction at which air commences to enter the soil, beginning with the largest pores at the exterior of the soil. The residual water content is the water content value at which a large increase in suction is required to remove additional water from the soil. An indication of the air entry value and the residual water content is shown in Fig. 23. Thermodynamic considerations show that the total suctions corresponding to zero water content is approximately 1 million kPa.

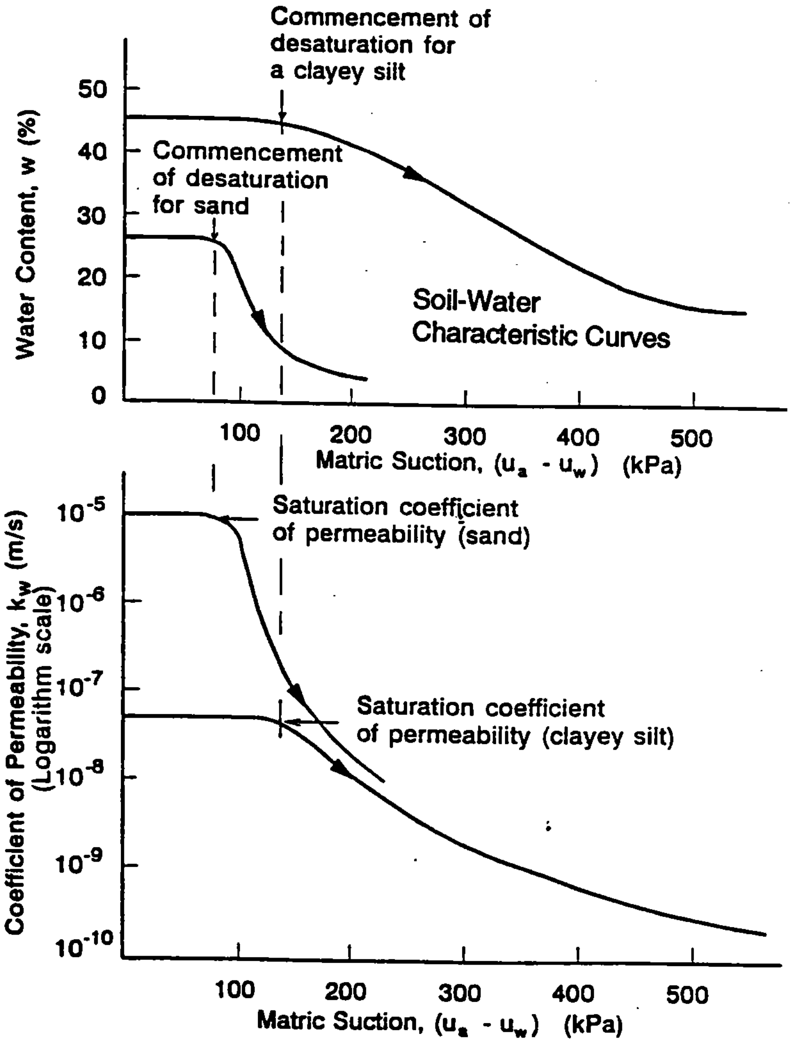


Figure 21 Relationship between soil-water characteristic curve and the coefficient of permeability for a sand and a clayey silt.

Equation for Soil-water characteristic Curves

Numerous equations have been proposed to characterize the soil-water characteristic curve. Fredlund and Xing (1994) proposed the following general equation for the soil-water characteristic curve,

$$[18] \quad \theta_w = \theta_s \left[\frac{1}{\ln\left[e + \left((u_a - u_w) / a \right)^n \right]} \right]^m$$

where :

θ_w = volumetric water content,

θ_s = volumetric water content at saturation,

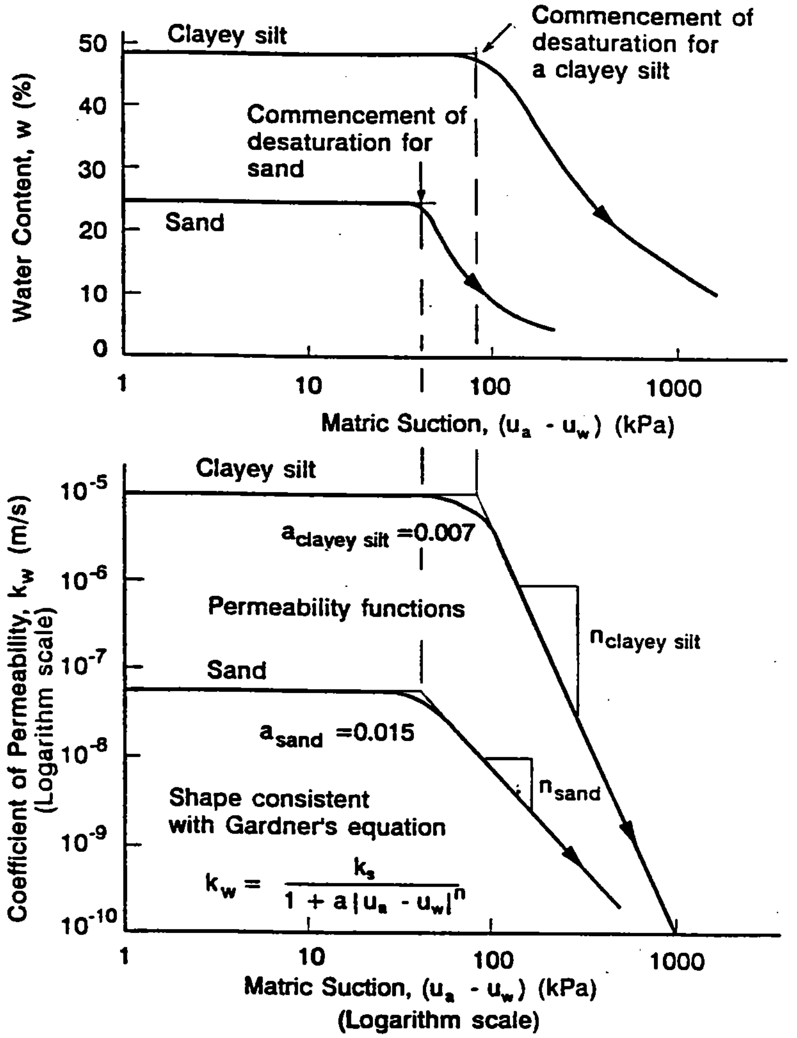


Figure 22 Typical Gardner's empirical permeability functions shown for a sand and a clayey silt.

e = 2.718....
$(u_a - u_w)$ = matric suction,
a = approximately the air entry value,
n = soil parameter related to the rate of desaturation,
m = soil parameter related to the residual water content

The upper limit of suction of 1 million kPa at zero water content can be incorporated into the above equation to give the following equation:

$$[19] \qquad \theta((u_a-u_w),a,n,m) = C(u_a-u_w)\frac{\theta_s}{\left\{ln\left[e+((u_a-u_w)/a)^n\right]\right\}^m}$$

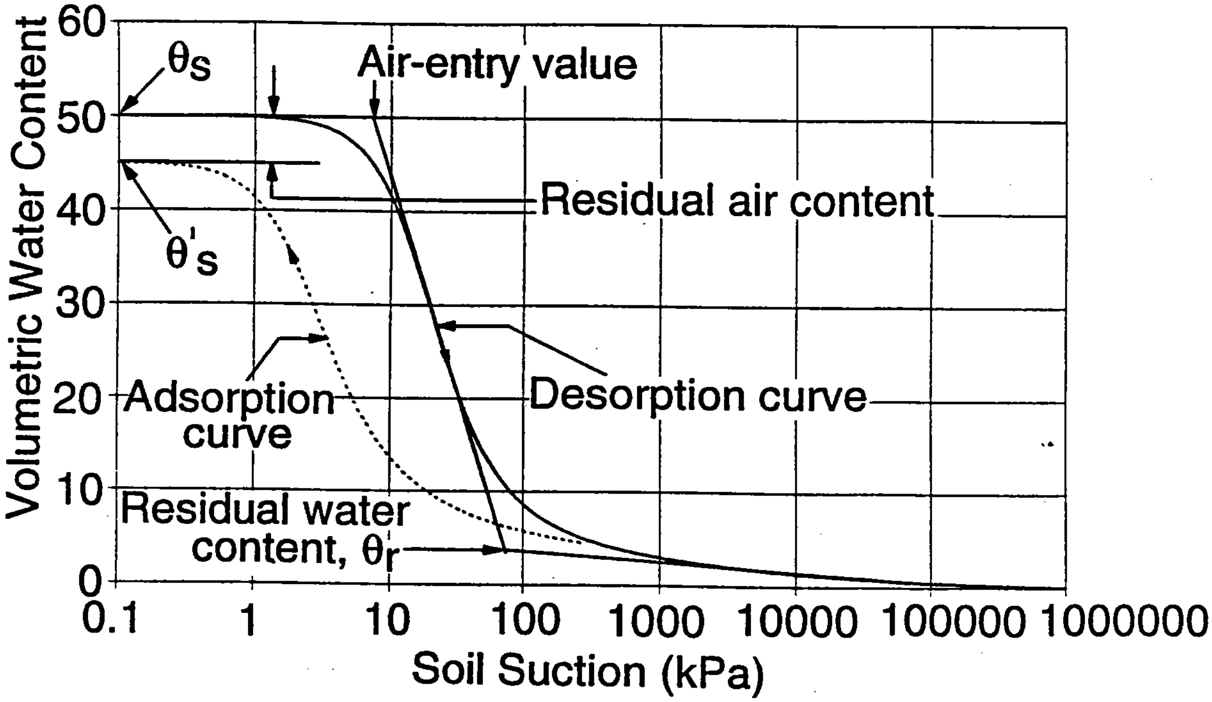


Figure 23 Typical soil-water characteristic curve for a silty soil.

where :

$$C(u_a - u_w) = 1 - \frac{\ln\left(1 + (u_a - u_w)/(u_a - u_w)_r\right)}{\ln\left[1 + \left(1\,000\,000/(u_a - u_w)_r\right)\right]}$$

Equation [19], along with the appropriate values of the relevant parameters, a, n and m, have been found to fit essentially all experimentally obtained soil-water characte-ristic curves.

In the following sections, it is shown how Eq. [19] can be used to obtain the shear strength function, the permeability function and the storage coefficient of unsaturated soils.

Relationship between soil-water characteristic curve and shear strength.

The slope, ϕ^b (i.e., slope of the shear strength versus matric suction curve) plotted against the matric suction bears a similar shape to the soil-water characteristic curve (Gan, Fredlund and Rahardjo, 1988). Figure 24 shows that the ϕ^b angle begins to deviate from the effective angle of internal friction, ϕ' as the soil desaturates at suctions greater than the air entry value of the soil. As the matric suction reaches a value corresponding to the residual water content, the ϕ^b angle appears to approach an angle near to zero degrees or it may even be negative.

The shear strength function can be obtained by integrating along the soil-water characteristic curve. The soil-water characteristic curve can be represented by Eqn. [19].

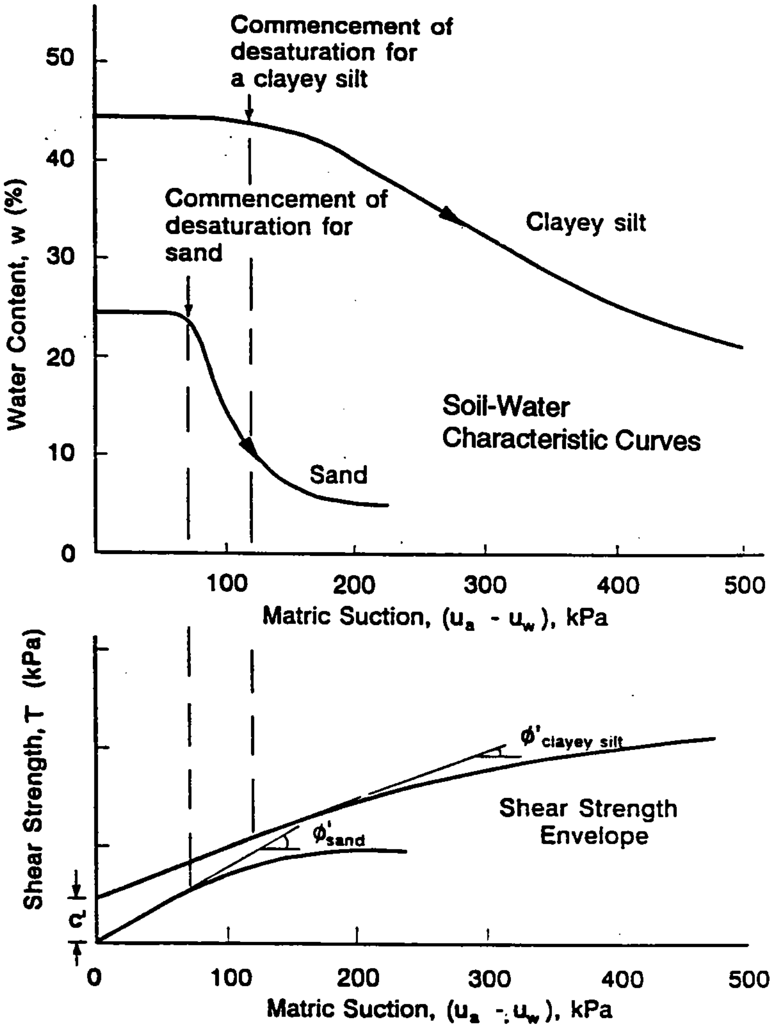


Figure 24 Relationship between soil-water characteristic curve and shear strength for a sand and a clayey silt.

The following shear strength expression as a function of matric suction and the effective angle of internal friction is obtained by Xing and Fredlund (1995).

$$\tau(u_a-u_w)=c'+(\sigma-u_a)\tan\phi'+\int_0^{u_a-u_w}\left[\Theta(u_a-u_w)\right]^p d(u_a-u_w) \qquad [20]$$

where : $\Theta(u_a-u_w)=\theta/\theta_s$, and

$\theta(u_a-u_w)$ = the volumetric water content at any suction, which can be represented by Eq. [19].

Relationship between soil-water characteristic curve and permeabililty.

The effective water conductive porosity for a given soil is a function of its water content. The water content of a given soil is in turn a function of the stress state of the soil. Therefore, there is a relationship between the hydraulic conductivity and the soil-water characteristic curve. This relationship was shown in Fig. 21.

Fredlund et al. (1994) derived the following expression for the permeability function of an unsaturated soil, when using the soil-water characteristic curve with matric suction on a logarithmic scale.

$$[21] \qquad k(u_a-u_w)=k_s\left[\frac{\displaystyle\int_{ln(u_a-u_w)}^{b}\frac{\theta\left(e^{ln(u_a-u_w)}\right)-\theta(u_a-u_w)}{e^{ln(u_a-u_w)}}\frac{\partial\theta}{\partial ln(u_a-u_w)}\left(e^{ln(u_a-u_w)}\right)d\,ln(u_a-u_w)}{\left[\displaystyle\int_{ln(u_a-u_w)_{aev}}^{b}\frac{\theta\left(e^{ln(u_a-u_w)}\right)-\theta_s}{e^{ln(u_a-u_w)}}\frac{\partial\theta}{\partial ln(u_a-u_w)}\left(e^{ln(u_a-u_w)}\right)d\,ln(u_a-u_w)\right]}\right]$$

where : k_s = saturated coefficient of permeability
b = ln (1 000 000) = 13.8155
e = 2.718....
$(u_a - u_w)_{aev}$ = air entry matric suction value

The equation for the volumetric water content, θ, as a fuction of matric suction, $(u_a - u_w)$, is given by Eq. [19].

Relationship between soil-water characteristic curve and storage coefficient

The storage coefficient, m_2^w, also known as the coefficient of water volume change with respect to change in matric suction, is the slope of the soil-water characteristic curve (Fig. 21). The storage coefficient, m_2^w, is obtained by differentiating the soil-water characteristic curve.

The equation for m_2^w is:

$$[22] \qquad m_2^w=\frac{\partial V_w}{\partial\,(u_a-u_w)}$$

If the volume of the soil remains a constant, the storage coefficient, m_2^w, can be expressed as:

[23] $$m_2^w = \frac{\partial \theta_w}{\partial (u_a - u_w)},$$

since $\theta_w = \frac{V_w}{V}$,

where: V_w = volume of water
V = total volume of soil.

The soil-water characteristic curve (i.e., the relationship between the volumetric water content, θ_w, and matric suction, $(u_a - u_w)$) represented by Eq. [19], can be differentiated with respect to matric suction, $(u_a - u_w)$ to give the storage coefficient, m_2^w,

[24] $$m_2^w = \frac{\partial \theta}{\partial (u_a - u_w)} = C'(u_a - u_w)\frac{\theta_s}{\left[ln\left(e + \left(\frac{(u_a - u_w)}{a}\right)^n\right)\right]^m} + C(u_a - u_w)\,D(u_a - u_w)$$

where :
e = 2.718....
θ_S = saturated volumetric water content

$$C'(u_a - u_w) = \frac{\left(\frac{1}{(u_a - u_w) + (u_a - u_w)_r}\right)}{\ln\left(1 + 1000000/(u_a - u_w)_r\right)}$$

and

$$D(u_a - u_w) = -mn\frac{\theta_s}{\left[ln\left(e + \left(\frac{(u_a - u_w)}{a}\right)^n\right)\right]^{m+1}}\frac{(u_a - u_w)^{n-1}}{e\,a^n + (u_a - u_w)^n}$$

SATURATED/UNSATURATED SOIL MECHANICS APPROACH TOWARDS GEOTECHNICAL ENGINEERING

In many routine problems, a saturated/unsaturated soil mechanics approach along with a consideration of the surface flux boundary conditions is found to be essential to the evaluation of the problem. A few examples are described in this section, under the classic soil mechanics categories of i.) seepage, ii.) shear strength and iii.) volume change, respectively.

Seepage Problems

1. Contamination of groundwater from chemical spills is of wide-spread serious concern. The movement of the contaminants from the ground surface to the groundwater table is a process which has to be analysed, taking into the consideration the continuum consisting of the atmosphere, the vadose zone and the saturated zone below the groundwater table. The vadose zone is the buffer providing protection to the groundwater. The movement of the contaminants through the vadose zone is controlled by the permeability and storage charcteristics of the soils in the vadose zone. In addition to bulk flow under gravity in the case of liquid contaminants, a conveyance or transport agent is required. The transport agent is usually water. The flux condition at the ground surface is therefore of importance in terms of the water that will be available for transport. The stress state of the soil in the vadose zone is affected by the flux occuring at the surface. The stress state (i.e., matric suction) affects the hydraulic conductivity of the soils.

2. Compacted clay covers have become a common solution to control water flow through waste management facilities (Fig. 25) The performance of the cover is largely controlled by the hydraulic conductivity and storage characteristics of the cover along with the surface flux to which the cover is subjected.

3. The movement of moisture in the entire region surrounding a waste containment area is often closely related to the hydraulic conductivity of the vadose zone. The mounding of the groundwater table below a waste containment area occurs in response to flow through unsaturated soils (Fig. 25). Although the soil below the waste containment may maintain negative pore-water pressures, there still will be flow in response to a hydraulic gradient, commensurate with a lower unsaturated hydraulic conductivity.

4. During the construction phase, much of the earth dam is consisted of compacted unsaturated soil. As the reservoir is filled, there will be water flow through the dam both in the negative and positive pore-water pressure regions. The permeability function which is a function of the negative pore-water pressure must be used in modelling the flow of water through the unsaturated zone.

Once the reservoir is filled, a steady state condition is achieved and most of the flow is through the saturated soil. However, one of the conditions which can trigger instability arises from the extended infiltration on the surface of the dam. In this case, water will infiltrate through the unsaturated zone above the phreatic line.

5. Expansive soils often cause distress to light structures as a result of the ingress of water into the soil. As the pore-water pressures increase, the soil expands. The flow may be through the fissures and cracks in the soil. This becomes a difficult unsaturated soil problem to analyze.

6. Long-term predictions with respect to wastes resulting from mining operations are strongly controlled by the assessment of the surface flux boundary conditions. This is particularly true for the case of "closure" or decommissioning of a mine.

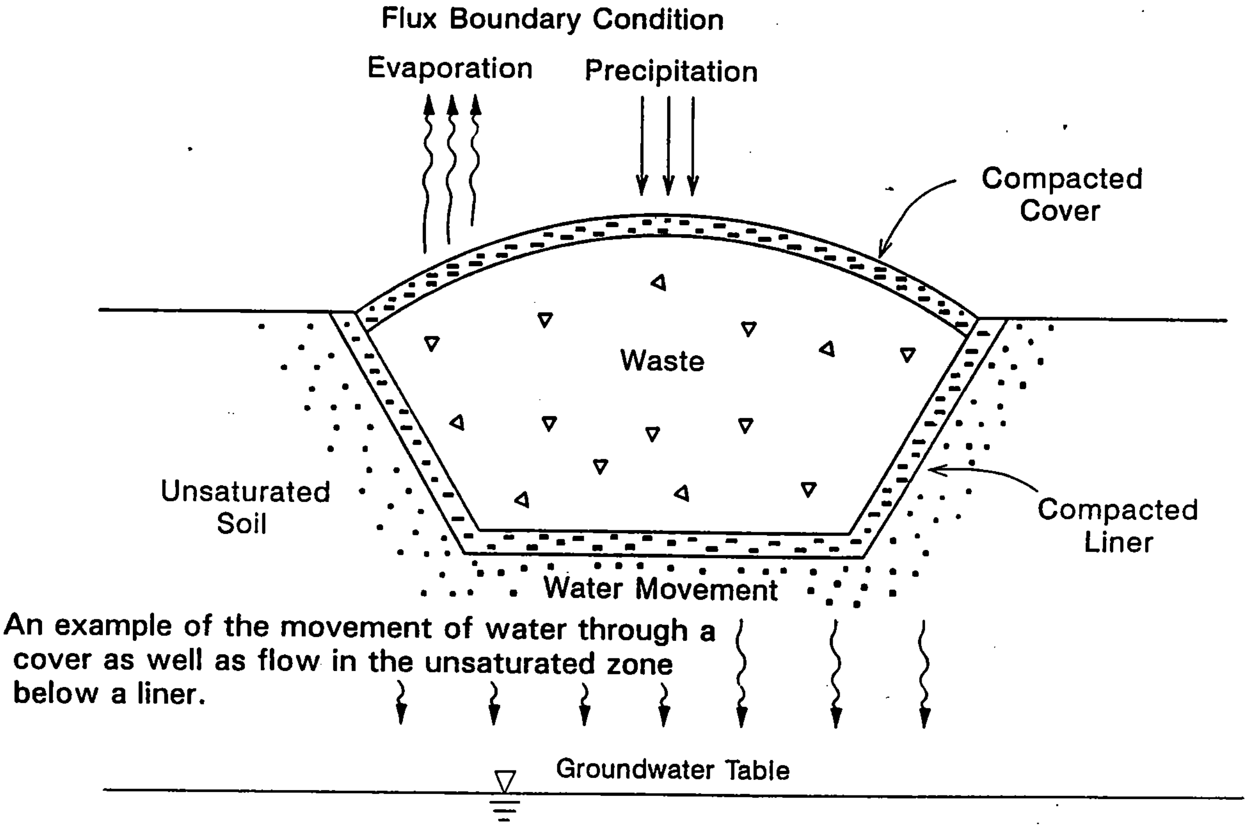


Figure 25 An example of the movement of water through a cover as well as flow in the unsaturated zone below a liner.

Shear Strength Problems

1. Natural slopes generally fail at some time following a high level of precipitation over a prolonged period of time. While the mechanism leading up to failure is well known, few attempts have been made to model the problem. The main reason appears to be related to the difficulties in modelling the flux boundary condition and the flow through the unsaturated zone.

2. The stability of loosely compacted fills can result in high velocity mass movements upon approaching saturation. The soil structure may experience collapse with the result that the load is transferred onto the water phase.

3. The stability of cuts or trenches for laying pipelines involves unsaturated soils. The costs associated with temporary bracing are high. Each year lives are lost due to collapse of the bracing arising from loss of shear strength of the unsaturated soil from wetting.

4. The assessment of stability of temporary excavations around construction sites is a difficult task. It is often left to the contractor to handle the problem. It has been observed in Southeast Asia that it is quite common practice for contractors to place a plastic membrane over a newly

excavated slope (Fig. 26). The slope may be part of an excavation for a foundation or part of a remediation (or landscaping). The plastic membrane ensures that a major portion of the rainfall will be shed to the bottom of the slope. In other words, the use of a plastic membrane is an attempt to maintain the negative pore-water pressures in the backslope. The practice of using plastic membranes in this manner has potential usage in other parts of the world.

5. The backfill material for an earth retaining structure should be a cohesionless material. However, many retaining structures are backfilled with cohesive materials which change volume and shear strength in response to the intake of water. The lateral pressures against the wall are a function of the shear strength of the unsaturated soil and the extent of wetting.

6. The bearing capacity of shallow footings is commonly based on the compressive strength of the soil. These strength measurements are often performed on soil specimens from above the groundwater table where the soil has negative pore-water pressures and may be fissured and unsaturated. The assumption is then made for design purposes that conditions in the future will remain similar. This may not be a realistic assumption.

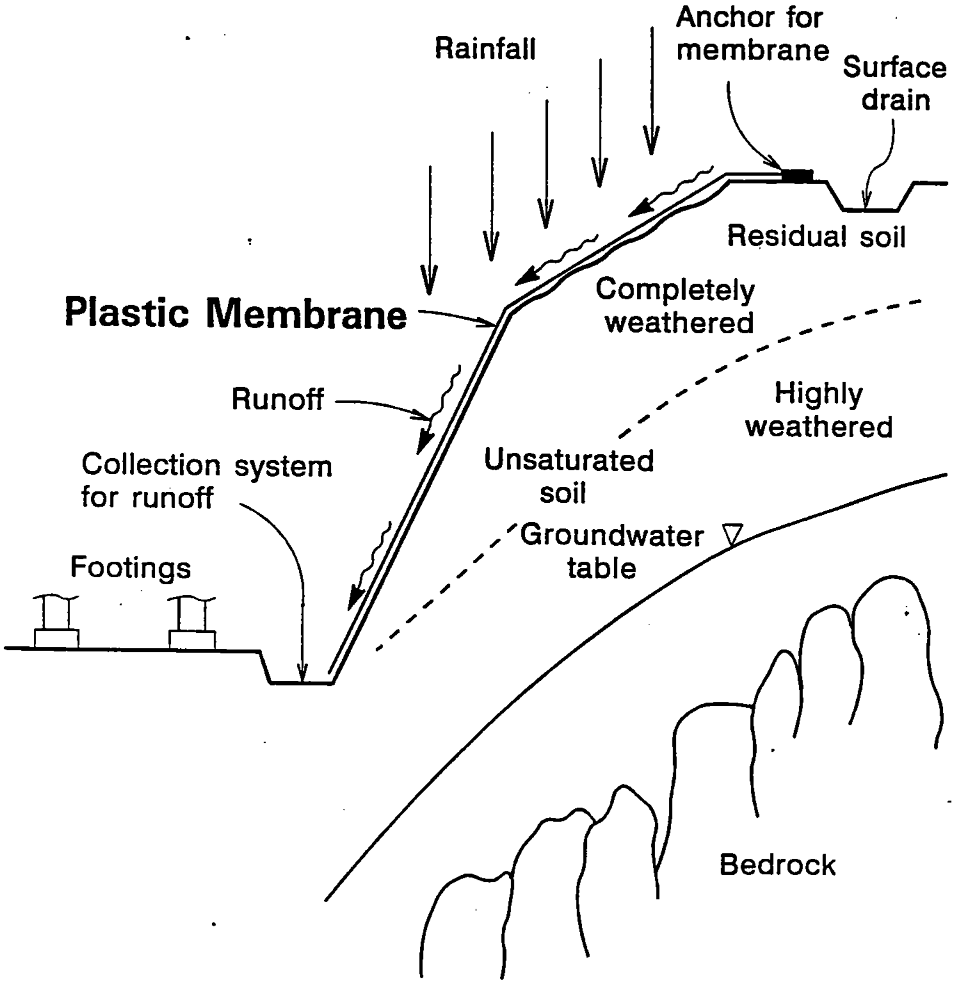


Figure 26 Example of the control of infiltration through the use of geomembranes.

Volume Change Problems

1. Volume changes in the soil below shallow footings generally take place in response to a moisture flux around the perimeter of the structure. While the mechanism involved is well understood, there is still a need for two- or three-dimensional numerical models to simulate these conditions.

2. In some countries, the shrinkage of high volume change soils under drying conditions poses a more serious problem than swelling. Shrinkage is the reverse of swelling in term of the stress state. However, the analyses and solutions for the two problems can be quite different.

3. The collapse of meta-stable-structured soils such as loess or poorly compacted silts and sand is a problem involving a volume decrease resulting from a decrease in suction.

4. The prediction of the depth of cracking is related to the volume change behavior of the soil. The depth of cracking influences earth pressure and slope stability solutions. An increase in suction increases the depth of cracking. A complete analysis of the problem is still not available.

5. There needs to be analysis for the volume change of compacted fills such as dams and embankments. The changes in volume of the unsaturated soil may be due to total stress or matric suction changes.

7. The cracking of covers for waste containment areas is an example of a geo-environmental problem for which a volume change analysis is required.

CONCLUSIONS

Many problems in geotechnical engineering involve unsaturated soils. In fact many engineering problems require the consideration of the continuum consisting of the ground surface through the vadose zone to the zone below the water table. It is important that engineers begin to analyze soils problems within one consistent theoretical context. There is a great need for seminars and workshops on unsaturated soil mechanics in order to transfer technology into the hands of practicing engineers. There is also a need for simplistic visual aids which would assist in the visualization of the complex concepts associated with unsaturated soil behavior. Some visual aids have been suggested to bring unsaturated soil behavior within the scope of Saturated/Unsaturated Soil Mechanics.

Problems involving unsaturated soils often have the appearances of being extremely complex. It is the hope of this paper to allay much of this fear. The paper has shown by way of a saturated/unsaturated soil mechanics approach that essentially the same theories and formulations established in classical soil mechanics are applicable to problems involving unsaturated soils through appropriate extensions to the established theory on saturated soil mechanics. The numerical computation involving unsaturated soils are generally more complex. However, the concepts are equivalents to those required for the understanding of classical soil mechanics.

One soil property which is recurent in dealing with unsaturated soils is the soil-water characteristic curve. The soil-water characterisric curve has emerged to be a key soil property which is of value in characterizing the behavior of unsaturated soil.

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